# Design Implementation With Noninteger Multiple-Height Cells for Improved Design Quality in Advanced Nodes

Sorin Adrian Dobre, Andrew B. Kahng, and Jiajia Li

Abstract-Standard-cell libraries can be developed with different cell heights (e.g., in FinFET technology, corresponding to different numbers of fins). Larger cell heights provide higher drive strengths, but at the cost of larger area and power consumption as well as pin capacitance. Cells with smaller heights are relatively smaller in area, but have weaker drive strengths and are more likely to suffer from routing congestion and pin accessibility issues. Existing design methodologies and tool flows are able to mix cells with different heights at the block level (i.e., each block contains cells with heights being integer multiples of a particular "single row" cell height). To our knowledge, no design methodology in the literature or in production mixes cells with different, noninteger multiple heights in a fine-grained manner. In this paper, we propose a novel physical design optimization flow to implement design blocks with mixed cell heights in a fine-grained manner. Our optimization resolves the "chickenand-egg" loop between floorplan site definition and the optimized choices of cell heights after placement with full comprehension of the constraints and costs of mixing cells of different heights (e.g., the "breaker cell" area overheads of row alignment between subblocks of 8 T and 12 T cell rows), our optimization achieves up to over 30% area and power reductions versus 12 T-only implementation while maintaining the same performance, and up to over 10% performance improvement along with power and area reductions versus 8 T-only implementation.

*Index Terms*—Digital integrated circuits, mixed cell height, physical design, placement algorithms.

## I. INTRODUCTION

**S** TANDARD cell-based implementation has been widely used for very large scale integration designs due to its relatively accurate abstraction and semi-regular layout. Cells are designed with different heights (e.g., different numbers of fins in FinFET node). Larger cell heights have higher drive strengths at the cost of larger area and power consumption as well as pin capacitance. Smaller cell heights result in relatively smaller area, but have weaker drive strengths and

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Fig. 1. Delay-area tradeoff of 8 T and 12 T buffers/inverters in 28 nm LP foundry libraries. Corner: [Slow (nMOS) slow (pMOS) process (SS), 0.95 V, 125 °C]. Load cap = FO4 + 20  $\mu$ m M3 wire.



Fig. 2. Post-synthesis netlist with mixed cell heights has significant area reduction compared to 12 T-only and 8 T-only netlists. Technology: 28 nm LP. Design: AES. Frequency: 1.5 GHz. Corner: (SS, 0.95 V, 125 °C). Total cell area and number of instances are normalized to those of the 8 T-only case. In the right figure, the solid bar indicates worst negative slack (WNS), and the shaded bar indicates TNS. Implementations with 12 T-only and mixed cell heights have no timing violations.

are more likely to suffer from routing congestion and pin accessibility issues. Although a cell with smaller height can be designed with large width to gain drive strength, the additional poly capacitance and metal capacitance can lead to area and power overheads as compared to a cell with larger height. Fig. 1 shows the delay and area tradeoff of buffers and inverters at foundry 28 nm linear programming (LP) technology.<sup>1</sup> Each circle represents a buffer or an inverter. In red are 12 T cells, and in blue are 8 T cells. As expected, 12 T cells can achieve smaller delay at the cost of larger area.

<sup>1</sup>The area values of buffers and inverters are from the liberty models in a foundry 28LP technology. We further estimate the intrinsic delay of a buffer or an inverter based on delay lookup tables from liberty models. Specifically, we assume fanout-of-four load capacitance together with an extra capacitance of 20  $\mu$ m M3 wire, and an initial input slew of 30 ps. We then iteratively feed the output slew of the cell to its input pin until the change of the output slew value is less than 1 ps. We then report the cell delay with the stabilized slew value in Fig. 1.

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Given that cells of different heights exhibit different tradeoffs among performance, power, and area, mixing cells of different heights in a design is able to provide a larger solution space and-up to some practical limit-improved design quality. Fig. 2 shows the post-synthesis area and timing comparison among implementations of an open-source design AES [17] with 12 T-only, 8 T-only, and mixed cell heights. Due to generally larger areas of cell instances (particularly those with low drive strengths), 12 T-only implementation tends to have larger design area. On the other hand, weak drive strengths of 8 T cells result in a large number of buffer insertions to meet timing constraints, which also increases design area.<sup>2</sup> We observe from the example that mixing cell heights achieves 14% and 18% area reduction at the post-synthesis stage versus the 12 T-only and 8 T-only implementations, respectively. Note that since the example only compares post-synthesis solutions, where layout effects are not considered, the example only shows a (motivating) estimation of the potential block-level benefits from mixing cell heights.

Motivated by the above observations, in this paper we propose to mix cell heights at the sub-block level (i.e., within a single P&R block) of physical implementation, to achieve improved design quality—specifically, tradeoffs of achievable performance, power, and area.<sup>3</sup> However, optimizing a design by mixing noninteger multiple cell heights is highly nontrivial. The challenges include the following.

- Current design methodologies and tool flows can only mix cells of different heights at the block level, i.e., each block of a design uses cells with a particular height, with fine-grained mixing not available with today's EDA tools.
- 2) There is a "chicken-and-egg" quandary: heights of cell rows are defined (in the placement site map) at the floorplan stage, but the optimized choices of cell heights are highly dependent on the placement outcome and timing constraints.
- 3) There are costs associated with mixing cells of different heights. For instance, "breaker cells" must be inserted for row alignment between sub-blocks of cell rows with different heights. We define a *breaker cell* as the space (i.e., placement and/or routing blockages) that must be inserted between the boundaries of regions of different cell heights.

The contributions of this paper are as follows.

- To our knowledge, we are the first in the literature to propose mixed cell-height implementation with noninteger multiple heights at the sub-block or subisland level in advanced nodes.
- We develop methodologies which can easily be integrated within existing physical design flow, using standard commercial tools, for mixed cell-height implementation.
- 3) We show that mixing 12 T and 8 T cells in a 28 nm LP foundry technology achieves up to over 30% area and power reductions versus 12 T-only implementation while

maintaining the same performance, and up to over 10% performance improvement along with power and area reductions versus 8 T-only implementation.

# II. RELATED WORKS

To our knowledge, there is no previous work on mixed cellheight design methodology that addresses noninteger multiple heights within a block. A recent work [12] optimizes cell placement with heights being integer multiples of a particular cell height. And, commercial placers have been capable of handling multicell row height cells (i.e., with heights that are integer multiples of a single-row height) for over two decades. However, placement of single-height, double-height, tripleheight, etc. cells does not require site map change (i.e., the floorplan rows and placement sites are fixed during the optimization), whereas in our problem, different noninteger cell heights require different row and site definitions. Therefore, the placement problem involving cells with multiple noninteger heights cannot be solved by existing optimizations that handle the integer-multiples problem.

Notwithstanding the above, the mixed cell-height placement problem bears some similarity to the problem of voltage island placement, in that both problems try to assign a certain attribute with different values (e.g., cell height or supply voltage) to standard-cell instances, in order to improve performance or reduce power. Further, both problems must comprehend a type of incurred cost (e.g., area overhead of breaker cells or insertion of level shifters) when performing the assignment. We therefore review exemplary works from the literature on voltage island placement.

Ching *et al.* [2] and Wu *et al.* [15] proposed partitioning methodologies to divide post-placement die area into regions which will be assigned to different supply voltages. The objectives are, respectively, to minimize the number of partitions and to handle nonrectangular partitions. Wu *et al.* [16] further considered timing constraints during the voltage assignment. However, the interaction with standard-cell placement is missing in all of these works. An improved optimization proposed in [14] performs incremental placement to move timing-critical cells out from the low-voltage regions by adjusting net weights and cell delays. Guo *et al.* [6] embedded their voltage-island-aware placement optimization to a partitioning-based placement algorithm to minimize the number of level shifters.

Although the voltage island placement problem has been well-studied in the previous literature, there is still no available solution for mixed cell-height design implementation due to the existence of chicken-and-egg loop between floorplan and cell height selection, additional layout constraints, and area impact of cell height choices.

## **III. PROBLEM FORMULATION**

Table I gives notations used in the following discussion.

We state the noninteger multiple-height cell placement problem as follows.<sup>4</sup>

Given a design (i.e., gate-level netlist), timing constraints, liberty and technology models for cell libraries with multiple track heights, and P&R block area and aspect ratio, place the

 $<sup>^{2}</sup>$ The larger total cell area of the 8 T-only netlist as compared to that of the 12 T-only netlist is due to tight timing constraints. We demonstrate in Section V that with loose timing constraints, 12 T-only implementations typically incur area overhead as compared to 8 T-only implementations.

<sup>&</sup>lt;sup>3</sup>Since today's P&R tools already support placement of integer multipleheight standard cells (e.g., [20]), this paper mainly focuses on implementation with noninteger multiple-height cells, which has not been addressed in the literature.

<sup>&</sup>lt;sup>4</sup>Since we focus on placement with noninteger multiple-height cells, in the following discussions "mixed cell heights" or "mixed heights" refers to "mixed noninteger multiple cell heights."

Term	Meaning					
$h_i$	available cell heights $h_0 \leq h_1 \leq \ldots \leq h_N$					
$(X^l, Y^b, X^r, Y^t)$	coordinates of block area (i.e., standard-cell placement region					
$P_j$	partition in which cells are of the same height, $(0 \le j \le M)$					
$(x_j^l, y_j^b, x_j^r, y_j^t)$	coordinates of partition $P_j$ , $(0 \le j \le M)$					
$H_j$	height of partition $P_j$ , $(0 \le j \le M)$					
$W_j$	width of partition $P_j$ , $(0 \le j \le M)$					
$t_j$	cell height corresponding to partition $P_j$ , $(0 \le j \le M)$					
$w_{site}$	placement site pitch (width)					
d	shift of cell rows in vertical direction to avoid cell overlap					



Fig. 3. Area cost of breaker cells.

design such that each cell instance is legally placed in row sites with corresponding height. The objective of the placement is to achieve minimum design power or area while maintaining the (same) target performance.

Additional layout constraints for mixed cell-height implementation applied in our studies below are as follows.<sup>5</sup>

- $C_1$ : To ensure manufacturability, each region of a particular cell height must have at least two cell rows.
- $C_2$ : Due to *N*-well sharing, each region of a particular cell height must have an even number of rows.
- $C_3$ : Every region with a particular cell height must align with the block's overall metal and poly track definitions.
- $C_4$ : The horizontal distance between two regions of different cell heights must be no less than four placement sites to honor the well-to-well spacing rule.
- C<sub>5</sub>: The minimum vertical distance between two partitions of different cell heights must ensure that the power/ground (P/G) rail of one cell does not encroach beyond the P/G rail of another cell. In other words, there must be a vertical gap to avoid P/G track alignment issue with mixed-cell heights. (Fig. 3 shows an example with M2 pitch = 64 nm, and P/G rail width = 48 nm and 64 nm for 8 T and 12 T cells, respectively. Although the P/G rail width difference between 8 T and 12 T cells is less than one M2 pitch, to align cells to routing tracks, the minimum *d* in the example is 64 nm.)



Fig. 4. Overall flow of our optimization. In the example, the maximum cut number (K) = 30. (a) Initial placement with mLEF, 8 T cell rows only. (b) Legalized placement with mLEF, 8 T cell rows only. (c) Optimized placement in updated floorplan with original LEF, mix of 12 T/8 T rows (with space for breaker cell).

 $C_6$ : Breaker cells must be inserted to ensure the minimum horizontal and vertical distances between two regions of different cell heights.

#### IV. METHODOLOGY

We now describe our optimization methodology for mixed cell-height implementation. The overall optimization flow is shown in Fig. 4. Given an input design (i.e., register-transfer level netlist) and timing constraints, we first synthesize it with liberty files of all available cell heights having been made available to the logic synthesis tool. To resolve the chickenand-egg loop between floorplan and cell height selection, we modify standard-cell library exchange format (LEF) files such that all cells have the same height (i.e., the minimum cell height among all the available heights), while maintaining the original area of each cell.<sup>6</sup> In the discussion below, we refer to such modified LEF files as modified LEF (mLEF). In this way, we break the chicken-and-egg loop and enable a commercial placement tool to "freely" place cells with timing-awareness. In other words, based on the synthesized netlist which contains various cell heights, we perform floorplanning with mLEF such that all cell rows have the same height (i.e., the minimum cell height among all the available cell heights). We then use a commercial placer (i.e., Innovus) to perform placement and post-placement optimization. Since we use the original liberty timing/power models and preserve the original area for each standard cell (although with modified aspect ratio for cells

<sup>&</sup>lt;sup>5</sup>Our proposed approaches given below transparently handle other values of the parameters that define these constraints (e.g., minimum number of cell rows in a given-height region, or minimum separation between two different-height regions, etc.).

<sup>&</sup>lt;sup>6</sup>In doing so, we round cell widths to the nearest whole site with no cell area reduction. E.g., given three libraries with heights 8 T, 9 T, and 12 T: 1) a 12 T, 6-site cell would be represented by an 8 T, 9-site cell and 2) a 9 T, 5-site cell would be represented by an 8 T, 6-site cell.

that originally had nonminimum height), this placement optimization is able to comprehend the tradeoff between timing constraints, power and area overheads. We emphasize that in the initial placement, we do not enforce any layout constraints or define regions for different cell heights, whereby the commercial placer is able to freely exploit the benefits from mixing cell heights. As a result, timing-critical cells tend to have larger heights (i.e., larger width with mLEF), while noncritical cells are smaller. An example initial placement solution of design AES is shown in Fig. 4(a), in which 12 T cells (with mLEF) are in red, and 8 T cells are in blue.

Based on the initial placement solution, we partition the block area into regions of particular cell heights with awareness of area cost due to breaker cells. Here, cell height refers to the original cell height as opposed to the cell height in mLEF. We then legalize the placement solution by: 1) displacement of cells (i.e., placement perturbation) and 2) swapping of cells across different heights (i.e., gate sizing). Here, we say that a placement solution is *legal* when each cell instance is placed in a region with the same height [e.g., as shown in Fig. 4(b)]. Once the placement solution is legal, we update the floorplan. Specifically, we perform refloorplanning such that the cell row height in each region is defined by the partitioning solution. We also insert space between regions with different cell heights to model the cost of breaker cells. We then map cells to the cell rows of the updated floorplan, using the original standard-cell LEF files. In the end, we perform clock tree synthesis and route the design [Fig. 4(c)]. In the following, Section IV-A describes our partitioning methodology, Section IV-B presents our placement legalization flow, and Section IV-C discusses cross-row displacement and cell mapping algorithms.

### A. Floorplan Partitioning and Region Definition

We perform slicing-based partitioning using dynamic programming to divide the block area into regions of particular cell heights. Algorithm 1 shows our partitioning procedure. We first evaluate the cost of each candidate partition, i.e.,  $cost(x^l, y^b, x^r, y^t, 0)$ , in which the fifth parameter indicates the number of cuts within the partition (line 1). If the height of a candidate partition is  $h_j$ , the cost of the partition is calculated as

$$\operatorname{cost} = \beta \cdot \sum_{h_i \neq h_i} \operatorname{area}_i + \lambda \cdot \Delta \operatorname{power} + \eta \cdot \Delta \operatorname{delay}$$
(1)

where area<sub>i</sub> is the total area of cells with height  $h_i$  located within the candidate partition;  $\Delta$ power is the estimated total cell power increase by swapping cells with original cell height  $h_i < h_j$  to height  $h_j^7$ ;  $\Delta$ delay is the estimated total cell delay increase by swapping timing-critical cells (i.e., cells with slack <20% of the clock period) with original height  $h_i > h_j$  to height  $h_j$ ; and  $\beta$ ,  $\lambda$ , and  $\eta$  are weighting factors.

Reducing the value of component  $\sum_{h_i \neq h_j} \operatorname{area}_i$  (i.e., the sum of areas of cells whose heights differ from the height of the partition) will help to minimize the cost of placement legalization (i.e., displacement and cell-height swapping) as well as the perturbation to the initial placement solution. Furthermore, to reduce the potential power and timing penalties due to legalization, we minimize the estimated cell power and delay increase within each candidate partition. More specifically, we first find

#### Algorithm 1 DP-Based Partitioning

```
1: calculate cost(x^l, y^b, x^r, y^t, 0)
\forall X^l \le x^l \le x^r \le X^r, Y^b \le y^b \le y^t \le Y^t
      for k := 1 to K do
3:
            for x^l := X^l to X^r - \Delta x do
4:
                  for y^b := Y^b to Y^t - \Delta y do
5:
                        for x^r := x^l + \Delta x to X^r do
                              for y^t \coloneqq y^b + \Delta y to Y^t do
6:
          \begin{aligned} & cost(x^{l}, y^{b}, x^{r}, y^{t}, y^{t}, k) = \min_{x^{l} \le x \le x^{tt}, y^{b} \le y \le y^{l}} (\\ & cost(x^{l}, y^{b}, x, y^{t}, k') + cost(x, y^{b}, x^{r}, y^{t}, k'') + 4 \cdot w_{site} \cdot (y^{t} - y^{b}), \\ & cost(x^{l}, y^{b}, x^{r}, y, k') + cost(x^{l}, y, x^{r}, y^{t}, k'') + d \cdot (x^{r} - x^{l}) \end{aligned}
 7:
                                                                 ) \forall k', k'' s.t. k' + k'' = k - 1
 8:
                               end for
9:
                        end for
10:
                    end for
11:
              end for
              if cost(X^l, Y^b, X^r, Y^t, k) \ge cost(X^l, Y^b, X^r, Y^t, k-1) then
12:
                   return cost(X^l, Y^b, X^r, Y^t, k-1)
13:
14:
             end if
15: end for
16: return cost(X^l, Y^b, X^r, Y^t, K)
```

the best candidate library cell (with height  $h_j$ ) for each cell instance (assume that the original height of the cell instance is  $h_i$ ), such that the best candidate library cell has the minimum cell power without any delay increase (resp. the minimum cell delay without any power penalty) if  $h_j > h_i$  (resp.  $h_j < h_i$ ). We then estimate  $\Delta$ power and  $\Delta$ delay values for each candidate partition accordingly.

Furthermore, we set the cost of a candidate partition to infinity if it violates any of the constraints (e.g., constraints  $C_1$ and  $C_2$ ) described in Section III. More specifically, a partition  $(x^l, y^b, x^r, y^t)$  with height  $h_j$  must satisfy

$$y^t - y^b \ge 2 \cdot h_j \tag{2}$$

$$\lfloor \frac{y^t - y^b}{2 \cdot h_j} \rfloor \cdot 2 \cdot h_j \cdot \left(x^r - x^l\right) \ge \left(y^t - y^b\right) \cdot \left(x^r - x^l\right) \cdot U_j.$$
(3)

Inequality (2) forces each partition to have at least two rows. Inequality (3) ensures that partitions in the updated floorplan, after rounding to an even number of rows per partition according to constraint  $C_2$ , have enough sites to place cells; here,  $U_j$  is the placement utilization within the partition. Finally, for each candidate partition, we set the height of the partition as the height which leads to the minimum cost function value.

Fig. 5 shows contour maps of power and delay costs, as well as partitioning solutions with various weighting factors of design AES.<sup>8</sup> The contour maps are plotted based on the sum of power or delay costs within each grid. As the value of  $\lambda$  decreases and the value of  $\eta$  increases, the area of 12 T regions and power both increase, while timing slack improves. In our experiments, we empirically set ( $\beta$ ,  $\lambda$ ,  $\eta$ ) as (1, 1, 1), (1, 0.1, 1), (1, 0.1, 0.1) and select the outcome with minimum power that satisfies timing constraints.

The heart of the dynamic programming recurrence (i.e., in determining the partitioning solution with minimum cost) is given in lines 2–16. We recursively search for the minimum-cost partitioning solution of a rectangular region with k cuts, and increase the value of k in each iteration up to a given maximum allowable number of cuts, K, which is a

<sup>&</sup>lt;sup>7</sup>Since dynamic power dominates in our testcases, we use the gate capacitance ratio between different cell heights to estimate the power increase.

<sup>&</sup>lt;sup>8</sup>We measure area, power, and delay in units of  $\mu$ m<sup>2</sup>,  $\mu$ W, and ps for cost function estimation. A small value of  $\beta$  will result in large perturbation of the initial placement, such that the power, area, and timing costs due to legalization can be high. On the other hand, a large value of  $\beta$  will limit the timing- and power-awareness in the partitioning optimization. We empirically use  $\beta = 1$  and vary the values of  $\lambda$  and  $\eta$  between 0 and 1 to explore the tradeoff between power and timing during partitioning optimization.



Fig. 5. (a) Contour map of power cost function. (b) Contour map of delay cost function. (c) Partitioning solution with  $\beta = 1$ ,  $\lambda = 0.8$ , and  $\eta = 0.2$ . (d) Partitioning solution with  $\beta = 1$ ,  $\lambda = 0.7$ , and  $\eta = 0.3$ . (e) Partitioning solution with  $\beta = 1$ ,  $\lambda = 0.6$ , and  $\eta = 0.4$ . Cell area, power, and delay are, respectively, measured in units of  $\mu m^2$ ,  $\mu W$ , and ps for cost estimation. In red are 12 T cells, and in blue are 8 T cells. Design: AES. Technology: 28 nm LP. By comprehending power and timing penalties, our partitioning optimization defines the height of regions with large power penalty as 8 T (e.g., region defined by black boundaries) and the height of regions with large timing penalty as 12 T (e.g., regions in yellow-dotted boxes).



6000 5000 3000 1000 0 0 1000 2000 3000 4000 5000 6000

Fig. 6. Examples of partitioning solutions for the AES testcase. In red are 12 T cells (with mLEF); and in blue are 8 T cells. Yellow lines are cuts. The cell height of a partition is marked on its side.  $\beta = 1$ ,  $\lambda = 0$ , and  $\eta = 0$ . (a) Cut number = 5 and cost = 4818  $\mu$ m<sup>2</sup>. (b) Cut number = 10 and cost = 4584  $\mu$ m<sup>2</sup>.

user-defined parameter.<sup>9</sup> Specifically, the for loop in line 2 sweeps the number of cuts. The for loops in lines 3–6 further enumerate all regions to optimize with a given minimum grid size (i.e.,  $\Delta x \times \Delta y$ ), where  $(x^l, y^b)$  and  $(x^r, y^t)$  are, respectively, the coordinates of the lower-left and upper-right corners of a region, and  $(X^l, Y^b)$  and  $(X^r, Y^t)$  are, respectively, the coordinates of the lower-left and upper-right corners of the design block. Fig. 6 and its caption tell us that the total cost within partitions reduces as the number of cuts increases.<sup>10</sup> However, the area cost of breaker cells increases with the number of cuts. We therefore sweep the number of cuts during our partitioning optimization and select the solution with the minimum total cost.

To find the best partitioning solution of a region  $(x^l, y^b, x^r, y^l)$  using exactly k cuts, we observe that such a solution can always be seen as a single "top-level" cut, along with the best solutions of the two subregions induced by that cut. Hence, to find the best k-cut solution, we enumerate all potential vertical and horizontal cuts of the region (where x and y in line 7, respectively, indicate the location of the vertical and horizontal cuts), and select the solution that minimizes the sum of the costs of the two separate parts (subregions)—with respective number of cuts k' and k'' satisfying k'+k''=k-1—plus the cost of the single vertical or horizontal top-level cut.

Fig. 7. Runtime of DP-based partitioning with number of grids (i.e.,  $M \times N$ ) varying from 400 to  $\sim$ 6000 using a single thread on a 2.5 GHz Intel Xeon server. The maximum cut number is fixed at 40.

Note that the proposed partitioning comprehends the area cost of breaker cells, for which width =  $4 \cdot w_{site}$  for a vertical cut, and height = d for a horizontal cut (line 7). For the example shown in Fig. 3, d must be larger than 64 nm. The procedure terminates when the cost does not decrease with an increased cut number (line 13), or the maximum cut number K is achieved (line 16). To improve the scalability, we divide the block area into  $M \times N$  grids (where M and N are also user-defined parameters), and perform the proposed partitioning method on these grids. The runtime complexity of the procedure is  $O((M + N)(M \cdot N \cdot K)^2)$ . Fig. 7 shows the empirical runtime of our dynamic programming (DP)-based partitioning. We observe that partitioning with no larger than  $50 \times 50$  grids and 40 cuts requires less than 10 min using a single thread on a 2.5 GHz Intel Xeon server. It must be noted that, as an initialization step, we traverse all cell instances in the design to estimate the potential cost of each grid. The corresponding runtime complexity of this initialization step is O(G), where G is the total number of cell instances. In practice, this initialization requires less than one minute for a design with 60 K instances.

# B. Timing-Aware Placement Legalization

Based on the partitioning solution, we perform iterative optimization to achieve a legal placement. Note that we still use mLEF at this optimization stage, but boundaries and cell heights of regions have been defined. We apply two knobs in our iterative heuristic: 1) displacement of a cell (e.g., moving a 12 T cell from an 8 T region to a 12 T region) and

<sup>&</sup>lt;sup>9</sup>In our experiments, we set *K* to a large value (e.g., 30 for a 100  $\mu$ m×100  $\mu$ m floorplan) in order to ensure good solution quality.

<sup>&</sup>lt;sup>10</sup>For clarify of illustration, we only consider the component of area cost in Fig. 6, which is indicated by the amount of red (12 T) or blue (8 T) area in the figure. Therefore, the costs are measured in units of  $\mu m^2$ .

	<b>hm 2</b> Heuristic to Legalize Placement
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1:	while there exists a cell with a different height than its partition do
2:	$list \leftarrow \emptyset$
3:	for all cell g with a different height than its partition do
4:	calculate cost function of g
5:	add g to list
6:	end for
7:	sort list in order of decreasing cost
8:	swap $cnt \leftarrow 0$
9:	for all $g \in list$ do
10:	apply displacement/swapping based on cost function
11:	incremental timing analysis
12:	if slack of $g < min(0, original slack of g)$    whitespace_of_grid < $\omega$ then
13:	undo change
14:	else
15:	++swap_cnt
16:	end if
17:	if $swap\_cnt \ge \gamma \cdot total\_gate\_count$ then
18:	apply ECOs in Innovus
19:	correlate internal timer with Innovus
20:	for all cell g in the design do
21:	downsize g
22:	incremental timing analysis
23:	if slack of $g < min(0, original slack of g)$ then
24:	undo change
25:	end if
26:	end for
27:	if WNS $\leq -\theta \cdot \text{clock}$ _period then
28:	fix maximum transition violations
29:	timing recovery
30:	apply ECOs in Innovus
31:	correlate internal timer with Innovus
32:	end if
33:	end if
34:	end for
35:	end while

2) cell-height swapping (e.g., assign an 8 T cell to a 12 T cell master in a 12 T region via gate sizing). Both of these knobs affect timing, and cell-height swapping also affects area. Thus, to ensure that the optimization does not lead to large design quality degradation, we evaluate the timing and area impacts of each potential move (one move is a cell displacement or a cell-height swap).

Because timing analysis with commercial P&R tools is typically slow, our optimization approach requires a relatively accurate and fast timing engine. We have developed an internal timing analysis engine (i.e., *internal timer*) to guide the optimization. Our internal timer estimates gate delay and slew at an output pin based on the liberty lookup tables. It further uses D2M [1] and PERI [11] models that, respectively, estimate wire delay and slew propagation along the interconnect. Wirelength change due to cell displacement is measured by net half-perimeter wire length (HPWL), and wire capacitance and resistance are scaled correspondingly. More specifically, we scale a given net's latest correlated (with the P&R tool) wire parasitics by a ratio  $WL_{cur}/WL_{corr}$ , where  $WL_{cur}$  is the net's current HPWL and WLcorr is the net's HPWL in the placement corresponding to the latest correlation with the P&R tool. This gives us an approximate estimation of updated (i.e., current) wire parasitics, and hence of the net's wire parasitic change. To correlate the internal timer with the P&R tool (line 19 in Algorithm 2), we update cell locations, sizes, and threshold voltage (VT) types in the P&R tool through a Tcl socket, and use the P&R tool to perform placement legalization, trial route, parasitic extraction, and timing update. We then update the cell locations, wire parasitics in our internal timer correspondingly. We apply a slack offset at each pin to match the internal timing slacks to those from the P&R tool following the general approach of [9] and [13].

To comprehend wire congestion effects, we add a penalty in the form of wire resistance and capacitance scaling, based



Fig. 8. Framework of our optimization.

on routing demand versus supply overflows within the bounding box of a given net. We estimate overflow based on the trial routing solution from Cadence Innovus Implementation System v16.1 [20]. More specifically, if the average horizontal (resp. vertical) routing congestion within the bounding box of a net is X%, we penalize the horizontal (resp. vertical) portion of HPWL by a multiplicative factor of  $(X\%-X_{th}\%)$ whenever  $X > X_{\text{th}}$ . Here,  $X_{\text{th}}\%$  is a threshold that we set to 95% based on separate studies. The value  $X_{\rm th}\% = 95\%$ is used in all experiments reported below.<sup>11</sup> To maintain the accuracy of our internal timer, we correlate timing slack, wire capacitance, and overflow information during the optimization through a Tcl socket with Cadence Innovus Implementation System v16.1 [20]. Fig. 8 shows our optimization framework. We believe that our internal timer approach most closely resembles that of the previous work [9]; however, our internal timer better comprehends the impact of cell displacement on timing by considering both wirelength change and routing congestion information.

Algorithm 2 describes our heuristic to legalize the placement. We first evaluate the cost (in terms of area and timing) of each potential move (i.e., cell displacement or swapping) (line 4). We consider cell displacement in eight directions (i.e., {N, S, E, W, NE, NW, SE, SW}) with the maximum movement distance of D ( $D = 15 \mu$ m in our experiments). The set of candidate cell displacements is similar to what is applied in the local optimization of [7]. For cell-height swapping, we consider candidate library cells whose heights match that of the partition. We use the cost function shown

$$cost = \alpha \cdot \frac{\max(0, -\Delta slack)}{\max(1ps, slack_{orig})} + (1 - \alpha) \cdot \frac{\max(0, \Delta area)}{\max(1\mu m^2, whitespace_{orig})}$$
(4)

where  $\Delta$ slack and  $\Delta$ area are, respectively, the timing slack and cell area changes due to displacement and/or swapping. slack<sub>orig</sub> and whitespace<sub>orig</sub> are the original timing slack of the cell and whitespace of the corresponding grid. We divide the block area into an  $M \times N$  mesh of grids. For each grid, we estimate whitespace based on placement utilization. The parameter  $\alpha$  is a weighting factor, which has an initial value of 0.5. We adaptively change the value of  $\alpha$  for each cell during the iterative optimization, such that when an attempt

<sup>&</sup>lt;sup>11</sup>For example, if the average horizontal congestion is 98%, we multiply the *x*-component of HPWL by 1.03 = 0.98/0.95.

leads to timing violation (resp. placement utilization violation), we increase (resp. decrease)  $\alpha$  of the cell by 1.5×.

The cost function (4) only considers area and timing impacts due to each move. In separate studies, we have also included input pin and wire capacitance, as well as leakage power, into our cost function for legalization moves. However, the resultant solutions show negligible improvement (e.g., <1%) in terms of power, area, and timing. This might be due to the positive correlations among area, pin capacitance, and leakage power. Furthermore, we adaptively change the value of  $\alpha$  during our optimization. As a result, we observe from our experiments that moves with small area and power costs are typically selected during the early steps. As the value of  $\alpha$ increases, moves with small delay penalties are selected during the late-stage legalization.

We sort all cells which have different height than their partition in decreasing order of cost, and apply moves to legalize the placement (line 7). When a move results in timing failure or violation of placement density, we undo the move (lines 12-13); here  $\omega$  is the required whitespace according to the area of breaker cells and maximum placement density constraints.<sup>12</sup> We rely on the commercial P&R tool (i.e., *Innovus*) to perform incremental placement legalization. By considering the placement density, we minimize the impact of potential displacement on the cells that are already legalized. To ensure the convergence of the flow (i.e., that optimization can lead to a legalized placement), we commit the move of a cell which has been visited F times, regardless of its impact on timing and area. We use F = 5 in our optimization.<sup>13</sup> In addition, we apply a form of Tabu search [5] during the optimization to increase the likelihood of finding feasible solutions for cells. Specifically, we record the latest three attempts and forbid these moves for the current move of optimization. During the optimization, we (re-)correlate our internal timer with Innovus in terms of timing slack/slew, cell location, wire parasitic, and routing, overflow after every  $\gamma \%$  of the total number of cells has been changed (lines 17–19), so that cost function terms will be estimated based on accurate timing and placement density information. We use  $\gamma = 2$  in our optimization.<sup>14</sup> We also include area recovery (lines 20-26) and timing recovery (lines 27-32) in our optimization to maintain timing and area quality. The parameter  $\theta$  is a threshold of slack violation that triggers timing recovery; we empirically set this to 0.15. Note that during the timing recovery, we perform backward (in which we downsize fanout cells) and forward (in which we upsize cells) maximum transition violation fixes, which enhance the timing recovery quality.

TABLE II User-Defined Parameters

Term	Meaning					
$\beta, \lambda, \eta$	weighting factors used in partitioning cost function					
K	maximum number of cuts					
$M \times N$	number of grids					
D	maximum displacement distance					
$\gamma$	parameter to determine frequency of timing correlation					
θ	slack violation tolerance threshold					
F	maximum number of visits of a cell before a move is applied					

We observe from our experimental results that the ratio between the number of cells being swapped and the number of cells being displaced ranges from 1.2 to 5.4. This ratio seems highly dependent on the partitioning solution, timing constraints, netlist structure, etc. For instance, fewer partitions and/or tighter timing constraints can lead to more swaps relative to displacements.

We list all the user-defined parameters applied during partitioning and placement legalization in Table II.

# C. Mapping From mLEF to Original LEF in Assigned Regions

As discussed above (e.g., in the context of Fig. 4), we use mLEF with adjusted aspect ratios for cell layouts during the initial placement, partitioning, and legalization stages, where all cells have the same height (i.e., the minimum cell height  $h_0$ ) but scaled cell widths. When the placement solution is legalized (i.e., each cell instance is placed in a region with the same height), we update the floorplan to insert cell rows according to the *actual* cell height of each partition. We also allocate space to model the area cost of breaker cells in the updated floorplan. Thus, there are two distinct optimizations that we apply—*cross-row cell displacement* and *cell mapping*—which we describe next.

Cells with the minimum height have their original layout aspect ratios in mLEF, and therefore no aspect ratio changes are needed to map these cells in the updated floorplan. In a partition with the minimum cell height, we only remove overlaps between cell instances and breaker cells. In other words, we perform *cross-row cell displacement* to ensure that the total cell width (i.e., total number of placement sites) in a row does not exceed the available number of placement sites with the existence of breaker cells (which are modeled as spaces in our experiments).

Algorithm 3 describes our cross-row cell displacement procedure. We perform four iterations of cross-row cell displacements. The first and the third iterations are top-to-bottom, traversing from the topmost row to the bottommost row, and optimizing one row at a time. The second and the fourth iterations optimize similarly, but in a bottom-to-top manner. Furthermore, in the first and second iterations, we move cells to adjacent rows only if there is enough space in the adjacent rows. In the third (resp. fourth) iteration, we force cells to move to the lower (resp. upper) adjacent row regardless of available space in the adjacent row. This strategy enables chained moves of cells across rows. In Algorithm 3, k is the row index, which ranges from 1 (the bottommost row) to R(the topmost row);  $W_i$  is the total cell width in the *i*th row;  $W_i^{\text{max}}$  is the maximum allowed cell width in the *i*th row; w(g)is the width of cell g;  $\Delta$  records the HPWL increase due to cross-row cell displacement; and function move(g, dir) moves

 $<sup>^{12}</sup>$ In our experiments, we set the maximum placement density of the entire block as the placement density from the initial placement plus 5%.

<sup>&</sup>lt;sup>13</sup>We observe in our experiments that the number of cells which have been visited six times without a feasible solution is quite small, e.g., less than 60 in a design with 15 K cells. Moreover, although such a move will cause density violation within a grid, the neighbor grids typically have enough whitespace for placement legalization (i.e., the placement density constraint of a partition can still be met). However, for a high-density design, the move of a cell must ensure that the placement density of a partition does not exceed the density constraint. Otherwise, a repartitioning step might be required.

 $<sup>^{14}</sup>$ In our experiments, each correlation of our internal timer with *Innovus* takes  ${\sim}20$  s on a design with 15 K instances and  ${\sim}42$  s on a design with 60 K instances. The number of correlations highly depends on the initial partitioning solution (i.e., number of cell instances to be legalized). We empirically observe from our experiments that the correlation takes  ${\sim}15\%$  of the total runtime of our optimization, which includes partitioning, placement legalization, and cell mapping.



Fig. 9. Example of cross-row cell displacement (an 8 T region located at the lower-left corner of the DMA block). (a) Before cross-row cell displacement. In blue are 8 T cells. In gray are 12 T cells. (b) After cross-row cell displacement. In green are breaker cells. Breaker cells introduce routing blockages on M1/M2, whereas ports (in yellow) are placed on M3 and above.

#### Algorithm 3 Cross-Row Cell Displacement

```
1: for k := 1 to 4 do
          for all i<sup>th</sup> row in the partition do
 2:
               while i^{th} row has capacity/overlap violation do
 3:
4:
                   \Delta_{min} \leftarrow +\infty; \quad g_{move} \leftarrow \emptyset
for all g \in i^{th} row do
 5:
6:
                        if i \neq 1 && (W_{i-1} + w(g) \le W_{i-1}^{max} || k == 3) then
 7:
                             \Delta \leftarrow HPWL increase by moving g to (i-1)^{th} row
 8:
                             if \Delta < \Delta_{min} then
 9:
                              \Delta_{min} \leftarrow \Delta; \quad g_{move} \leftarrow g; \quad dir = \text{down}
end if
10:
11:
                         end if
                         if i \neq R && (W_{i+1} + w(g) \leq W_{i+1}^{max} || k == 4) then
12:
13:
                                 \leftarrow HPWL increase by moving g to (i+1)^{th} row
14:
                              if \Delta < \Delta_{min} then
15:
                                   \Delta_{min} \leftarrow \Delta; g_{move} \leftarrow g; dir = up
16:
                              end if
17:
                         end if
18:
                    end for
19:
                    if g_{move} == \emptyset then
20:
                         break
21:
                    else
22
                         move(g<sub>move</sub>, dir)
23:
                    end if
24:
               end while
25:
          end for
26: end for
```

cell g to the row that is adjacent in direction dir. After the cross-row cell displacements, we perform single-row incremental placement optimization using dynamic programming to further reduce wirelength.<sup>15</sup> Fig. 9 shows an example of our cross-row displacement optimization.

On the other hand, cells originally having large height (i.e., larger than the minimum cell height) become shorter and wider in mLEF. We must recover the original aspect ratio of cell layouts in the updated cell rows with actual cell heights. For example, assume that there are 20 10 T cells uniformly placed in five 8 T cell rows (i.e., as a  $5 \times 4$  mesh). To update the floorplan, we maintain the same partition area and place cell rows according to the height of the partition. We therefore have four 10 T cell rows. Given that the layout of these 10 T cells (with the same cell area) are scaled back to their original height with a reduced cell width, five cells now can fit into one row in the updated floorplan. The mapped cell placement becomes a  $4 \times 5$  mesh. Note that in this mapping procedure, the region outlines are fixed, and we only update the cell row height within each region [as illustrated in Figs. 4(b)-(c) and 11]. As shown in the example, cell mapping in the updated floorplan

<sup>15</sup>Our dynamic programming formulation is the same as the "minimum HPWL" formulation in [10]. Formulation details are given in [10].



Fig. 10. Illustration of graph embedding (a) from [4] and (b) for proposed cell mapping. Vertical connections are not shown.

can be viewed as embedding a graph to another graph with a different aspect ratio (e.g., embed a  $5 \times 4$  mesh to a  $4 \times 5$  mesh). We therefore revisit the graph embedding literature.

Ellis [4] showed that to embed a 2-D mesh of size  $w \times h$ (with unit distance between every two adjacent nodes in both horizontal and vertical directions) to another 2-D mesh of size  $w' \times h'$ , where w' < w and h' is the smallest integer satisfying  $w' \cdot h' \ge w \cdot h$ , if (w/w') is no larger than 2, the maximum wirelength of a two-pin net (in Manhattan distance) in the embedded graph is no more than two units. An example with (w/w') = (5/4) is shown in Fig. 10(a): the wirelength of each connection in the original graph is one, and the maximum wirelength in the embedded graph (i.e., the diagonal connection) is two. Note that our optimization of cell mapping differs from [4], in that [4] varies the area of the graph (i.e., mesh) while our optimization assumes a fixed mesh area (i.e., area of a partition).

Following the discussions in [4], we can show that if we map a 2-D-mesh placement with cell height  $h_0$ , in which all cells have the same cell area, to another 2-D-mesh placement with cell height  $h_1$ , the maximum wirelength scaling of a mesh edge (i.e., two-pin net) according to the mapping is no more than  $(h_0/h_1) + (h_1/h_0)$ .<sup>16</sup> Fig. 10(b) shows an example with 10 T and 8 T cells. Assuming unit wirelength for each two-pin connection between any horizontally or vertically adjacent cells in the original 2-D-mesh placement, the maximum wirelength increase is 1.05.

Inspired by the graph-embedding theory, we propose an approach to map cells to cell rows with original cell heights for general cases, in which cells have different widths and are not necessarily placed in a 2-D mesh. Algorithm 4 shows our procedure to map cells from an initial floorplan with Rrows of height  $h_0$  to an updated floorplan with R' rows of height  $h_i$ . We first estimate the average total cell width of each cell row in the updated floorplan (line 1), in which gis a cell in partition  $P_j$ ; w(g) is the actual width of cell g corresponding to height  $h_i$ . We then store cells in the *i*th row from the initial floorplan into list<sub>1</sub> and sort them by increasing order of their X-coordinates (lines 5-6). For each cell on the  $(i+1)^{\text{th}}$  row of the initial floorplan, we estimate the wirelength (i.e., HPWL) difference between the case of assigning the cell to the (i')th row of the updated floorplan versus the case of assigning the cell to the (i' + 1)th row of the updated floorplan (lines 9-12). We then sort the cells on the (i + 1)th row of the initial floorplan by the corresponding delta wirelength

<sup>&</sup>lt;sup>16</sup>Proof details are given in [4].

#### Algorithm 4 Cell Mapping

1:	$W_{avg} = \left(\sum_{g \in P_i} w(g)\right) / R'$
	i=1
	i = 1 $list' \leftarrow$ cells in $i^{th}$ row from the initial floorplan
	for $i' \coloneqq 1$ to $R'$ do
4. 5:	
6:	
	5
7:	
8:	
9:	
10:	$\Delta(g) \leftarrow HPWL\_diff(g, i', i'+1)$
11:	list'.push(g)
12:	end for
13:	sort <i>list'</i> in order of increasing $\Delta(g)$
14:	while $list' \neq \emptyset$ && $W \leq Min(1.05 \cdot W_{avg}, W_{i'}^{max})$ do
15:	$g \leftarrow list'.pop()$
16:	$list_2.push(g)$
17:	$W \leftarrow W + w(g)$
18:	end while
19:	sort list <sub>2</sub> in order of increasing cells' X-coordinate
20:	
21:	++i
22:	<i>list'</i> $\leftarrow$ cells in <i>i</i> <sup>th</sup> row from the initial floorplan
23:	
24:	$DPPlace(i', list_1, list_2)$
	end for

values (line 13). We iteratively add these cells to list<sub>2</sub> until the total width of cells in list<sub>1</sub> and list<sub>2</sub> exceeds  $1.05 \times$  of the average total cell width of each row  $(W_{avg})$  or the maximum allowed total width in the (i')th row  $(W_{i'}^{max})$ , where the width of breaker cells is considered) (lines 14-18). If all cells from list' are added to list<sub>1</sub> and list<sub>2</sub>, we fill list' with cells from the next row in the initial floorplan (lines 21-22). Finally, we use dynamic programming to order and place cells from list<sub>1</sub> and list<sub>2</sub> onto the (i')th row in the updated floorplan. In summary, lines 1–23 of Algorithm 4 determine Y-coordinates of cells and optimize the vertical component of HPWL; while the  $DPPlacer(i', list_1, list_2)$  further minimizes the horizontal component of HPWL. As an improvement to [8] which also uses dynamic programming to optimally order two rows of cells into a single row with minimized wirelength, our formulation also determines the optimal cell locations. The recurrence relation in our dynamic programming optimization is

$$sol(i, j, k) = Min \begin{cases} sol(i, j, k - 1) \\ sol(i - 1, j, k - w(g_i)) + cost(g_i, k) \\ sol(i, j - 1, k - w(g_i)) + cost(g_i, k) \end{cases}$$

where sol(i, j, k) is the wirelength corresponding to the optimal placement solution of the first *i* cells in list<sub>1</sub> and the first *j* cells in list<sub>2</sub> within the first *k* placement sites in the updated cell row;  $g_i$  is the *i*th cell in list<sub>1</sub>;  $g_j$  is the *j*th cell in list<sub>2</sub>; w(g) is cell width (i.e., in terms of the number of sites) of *g*; and cost(g, k) is the HPWL increase by allocating *g* (i.e., right edge of *g*) at the *k*th placement site.

Fig. 11 shows an example of our cell mapping optimization. With a fixed region outline, we map 21 rows of 12 T cells in 8 T height (i.e., using mLEF) to 14 rows of 12 T cells in 12 T height. Fig. 11 also shows inserted breaker cells around the 12 T region.

Fig. 12 shows the optimized wirelength (i.e., HPWL) comparison between our proposed method (using dynamic programming) versus a greedy method proposed in [3]. We observe that our proposed optimization achieves up to 16% wirelength reduction compared to the greedy method. Furthermore, it is obvious that the proposed algorithm can achieve the mapping solution or a solution with the same total wirelength shown in Fig. 10(b). We note that the procedure



Fig. 11. Example of cell mapping (a 12 T region located at upper-right corner of the block). (a) Before mapping optimization. 12 T cells (in red) are of 8 T cell height. In gray are 8 T cells. (b) After mapping optimization. 12 T cells (in red) are of 12 T cell heights. (8 T cells are moved to neighboring 8 T regions.) In green are breaker cells. The mapping of five specific cell instances is shown by labels 1-5.



Fig. 12. Wirelength comparison between our dynamic programming-based optimization versus a greedy optimization in [3]. Wirelength values are normalized to the wirelength before cell mapping.

TABLE III Benchmarks

Design	#Instances	#Flip-flops	Clock period range
AES	~16K	530	650ps – 800ps
DES	$\sim 23 \text{K}$	1984	600ps – 750ps
JPEG	$\sim 60 \text{K}$	4512	750ps – 900ps
MPEG	~16K	3193	550ps – 700ps

described in Algorithm 4 only applies to the case, where the ratio between  $h_j$  and  $h_0$  is no larger than two. We can easily extend our mapping procedure to address cases with height ratio greater than two by extending our dynamic programming formulation to optimize more than two lists of cells.

#### V. EXPERIMENTAL RESULTS

We perform experiments in a 28 nm LP foundry technology with dual-VT libraries, 0.95 V nominal supply voltage, and cell height choices 12 T and 8 T. To confirm that our optimization can perform a fine-grained mixed cell-height implementation, we select four design blocks (AES, DES, JPEG, MPEG) from the OpenCores [17] website. Parameters of these four testcases are shown in Table III. For each design, we determine a range of clock periods starting from a clock period with relative loose timing constraint, up to the clock period at which the 8 T-only implementation shows setup timing violations. These designs are synthesized using *Synopsys Design Compiler vI-2013.12-SP3* [18] and then placed and routed using *Cadence Innovus Implementation System v16.1* [20].



Fig. 13. Inserted space on the boundaries between 12 T and 8 T regions to model the cost of breaker cells.

In our experiments, we set the gate density at the floorplan stage as 60% and the aspect ratio of the floorplan as 1. We use the same commands and options for placement, postplacement optimization, clock tree synthesis (CTS), post-CTS optimization, routing and post-routing optimization for singleheight and mixed-height cases. We, respectively, use Cadence Innovus Implementation System and Synopsys PrimeTime-PX vH-2013.06-SP3 (PT-PX) [19] for timing and power analysis at the post-routing stage and wire parasitics (standard parasitic exchange format) obtained from Innovus. We use Synopsys PrimeTime vH-2013.06-SP2 [19] to search for the minimum supply voltage that satisfies a given frequency target. Our optimization flow is implemented in C++. Functions used in P&R tools and the socket between our optimizer and the P&R tool are implemented in Tcl. We conduct our experiments on a 2.5 GHz Intel Xeon server.

Modeling Breaker Cell Costs: The placement site pitch (width) and the M2 metal pitch in the 28 nm LP technology that we use are, respectively, 0.136  $\mu$ m and 0.1  $\mu$ m. Based on the discussion in Section III, the horizontal and vertical shifts between any 8 T and 12 T regions must be no less than 0.544  $\mu$ m and 0.1  $\mu$ m, respectively. In our experiments, we shift cell rows by 0.8  $\mu$ m in the vertical direction between any 12 T and 8 T regions (Fig. 13).<sup>17</sup> We also insert placement and routing blockages correspondingly. In FinFET technology, layout constraints for mixed-height design become more complicated (e.g., fin alignment). To evaluate the impact of breaker cell cost, we perform experiments with up to 3.5X area cost of the breaker cell in 28 nm technology. Results in Fig. 14 show small performance and power degradation when breaker cell area is  $< \sim 2X$ . When breaker cell area increases beyond  $\sim$ 2X, timing violation increases due to placement and routing congestion.

#### A. Performance-Area Tradeoff Comparison

We implement our benchmark designs using our proposed flow with mixed 8 T/12 T cells. We also perform conventional synthesis, placement, clock tree synthesis, and routing (SP&R) with 12 T-only cells and 8 T-only cells for comparison. The designs are implemented with clock periods shown in Table III. We use the nominal voltage 0.95 V at (SS, 125 °C) corner for design implementation and



Fig. 14. Results with different breaker cell area cost values. Design: AES. Frequency: 1.5 GHz.

timing analysis. Table IV shows our experimental results, in which the clock period is the clock period used for implementation and each benchmark design is implemented in four clock periods. Total power values are reported at the signoff frequency. We divide the block area of each design into grids of size around 6  $\mu$ m × 6  $\mu$ m for partitioning (i.e.,  $\Delta x = 6 \mu$ m and  $\Delta y = 6 \mu$ m) as illustrated in Section IV-A, and for placement density constraint evaluation as illustrated in Section IV-B. Fig. 15 further shows the Pareto curves illustrating tradeoffs between performance and area of implemented designs at the post-routing stage, where the frequency given is the maximum achievable operating frequency.

Results show that by mixing 8 T and 12 T cells, our optimization achieves significant area reduction (e.g., over 20% on design AES) compared to designs with only single-height cells, especially for comparison to 12 T-only designs (e.g., over 30% area reduction on design AES).<sup>18</sup> This is because mixed cell heights provide a wider range of tradeoff between performance and area such that 8 T cells are applied to timing paths with large slacks for area reduction, and 12 T cells are used in timing-critical paths to meet timing constraints. Furthermore, with loose timing constraints, the area benefit of mixed cell-height designs over 8 T-only designs reduces. On the other hand, mixed cell heights also have similar or even higher maximum achievable performance compared to designs with only single-height cells. For instance, we observe up to 13% performance improvement from mixed cell heights over 8 T-only designs (i.e., on design AES). This is because the maximum achievable performance of an 8 T-only design is limited by the weak drive strengths of 8 T cells. Moreover, mixed cell heights are able to have more compact area, which reduces wire capacitance, as well as smaller pin capacitance (i.e., by using 8 T cells) on nontiming critical fanouts of a timing-critical driver (which is typically a 12 T cell) compared to 12 T-only designs.

Experimental results also show that our optimization with mixed cell heights offers comparable routed wirelength compared to 8 T-only designs and smaller routed wirelength

 $<sup>^{17}</sup>$ Due to the lack of mixed-height implementation flow in commercial P&R tools, we implement a mixed-height design using a multiple power domain flow in *Innovus* [20], where regions with different cell heights are defined as different power domains. Such a flow allows different cell row heights within a design block. Further, due to the limitation of the commercial P&R tools, we insert space between regions (by defining the "minGap" attribute) to model the breaker cell area cost. As a result, cell rows must be aligned and the minimum vertical shift is 0.8  $\mu$ m instead of 0.1  $\mu$ m.

<sup>&</sup>lt;sup>18</sup>Note that our mixed-height optimization on design AES (clock period = 800 ps) results in a 8 T-only design, but with 27% area reduction compared to that of the 8 T-only implementation. This might be due to pessimism and a large number of inserted buffers during the synthesis stage of the 8 T-only flow. Specifically, at the post-synthesis and initial placement stages, the mixed-height design (which contains 18% 12 T cells) achieves 5% area reduction and 7% power reduction compared to the 8 T-only design. Moreover, our partitioning solution swaps all 12 T cells in the mixed-height design to 8 T cells, which results in 25% load pin capacitance reduction on average for each cell, and hence further reductions of area and power (but without timing violation due to the loose timing constraints).

TABLE IV PARAMETERS AND RESULTS OF IMPLEMENTED DESIGNS

Flow	12T	8T	mix	12T	8T	mix	12T	8T	mix	12T	8T	mix
Design (clk period)		AES (650ps)			DES (600ps	)		JPEG (750ps)		N	MPEG (550ps	3)
#Instances	14466	16056	14967	19896	23699	24738	52887	60137	57756	14117	15709	14820
12T/8T	14466/0	0/16056	14185/782	19896/0	0/23699	471/24267	52887/0	0/60137	4304/53452	14117/0	0/15709	1261/13559
LVT/RVT	11090/3376		2835/12132	10730/9166	18991/4708		20844/32043		20541/37215	3655/10462	7342/8367	2800/12020
Setup WNS (ps)	-54	-119	1	-0	-18	1	-0	-20	-0	-10	-143	2
Setup TNS (ns)	-2.755	-18.963	0.000	-0.000	-1.083	0.000	-0.000	-2.741	-0.000	-0.107	-18.861	0.000
#Hold violations	0	0	0.000	-0.000	-1.005	0.000	0	0	-0.000	-0.107	0	0.000
			-	-		-	-	-		-		-
Area $(\mu m^2)$	15481	13582	12190	20470	18218	15598	59998	49156	41703	21055	15494	13730
Utilization	85%	89%	81%	61%	69%	56%	63%	66%	56%	62%	70%	63%
WL (mm)	160	151	157	183	173	177	535	512	510	150	148	152
Leakage (mW)	0.489	0.366	0.158	0.346	0.385	0.203	1.017	0.959	0.511	0.216	0.206	0.090
Total power (mW)	35.0	30.0	25.7	57.2	52.9	44.9	67.1	58.6	51.4	34.4	30.2	27.7
Clock area $(\mu m^2)$	15	13	12	53	41	39	158	131	112	102	167	64
Clock WL (µm)	2017	1841	1463	8050	6834	6490	23977	19549	17912	11678	10620	10470
Runtime (min)	146	147	224	113	140	257	237	265	514	57	91	163
		AES (700ps)			DES (650ps			JPEG (800ps)		   N	MPEG (600ps	۱ <u>ــــــــــــــــــــــــــــــــــــ</u>
Design (clk period)				10504			54420		56004			
#Instances	14126	15688	14594	18504	22708	22321	54439	57310	56084	13776	15957	13655
12T/8T	14126/0	0/15688	437/14157	18504/0	0/22708	1692/20629	54439/0	0/57310	106/55978	13776/0	0/15957	614/13041
LVT/RVT	8361/5765	13432/2256	6226/8368	8905/9599	16021/6687	7675/14646	20489/33950		19556/36528		5217/10740	
Setup WNS (ps)	-3	-60	1	-0	-13	0	-2	-8	-0	1	-82	-5
Setup TNS (ns)	-0.009	-7.452	0.000	-0.000	-0.367	0.000	-0.005	-0.182	-0.000	0.000	-8.482	-0.048
#Hold violations	0	0	0	0	0	0	0	0	0	0	0	0
Area $(\mu m^2)$	14021	13241	8755	19586	16186	14539	59529	44998	41240	20171	14849	13268
Utilization	72%	93%	62%	62%	65%	59%	66%	64%	59%	63%	68%	62%
WL (mm)	166	141	135	180	159	158	518	477	468	147	139	136
Leakage (mW)	0.387	0.350	0.128	0.290	0.302	0.144	1.038	0.760	0.516	0.164	0.153	0.081
Total power (mW)	29.4	27.5	17.7	51.0	44.6	39.1	59.6	50.9	46.3	30.9	26.4	24.9
Clock area $(\mu m^2)$	15	13	11	49	35	36	109	100	93	81	102	65
Clock area $(\mu m)$ Clock WL $(\mu m)$	2079	1915	1363	7823	6107	6019	21308	22895	16612	11631	11270	9724
Runtime (min)	136	1515	1303	91	151	176	21308	310	495	37	65	107
				91				1		1		
Design (clk period) #Instances	13805	AES (750ps) 14981	14122	18123	DES (700ps 22206	) 21782	52401	JPEG (850ps) 58055	57304	13421	MPEG (650ps 14490	13478
		14201										
	12805/0	0/1/081	282/12840	18122/0				0/58055				
12T/8T	13805/0	0/14981	282/13840	18123/0	0/22206	842/20940	52401/0	0/58055	418/56886	13421/0	0/14490	1088/12390
LVT/RVT	6844/6961	11385/3596	4013/10109	7677/10446	0/22206 14688/7518	842/20940 5899/15883	52401/0 18120/34281	25312/32743	418/56886 15373/41931	13421/0 2144/11277	0/14490 5181/9309	1088/12390 1320/12158
LVT/RVT Setup WNS (ps)	6844/6961 -3	11385/3596 -24	4013/10109 3	7677/10446 0	0/22206 14688/7518 -8	842/20940 5899/15883 -0	52401/0 18120/34281 0	25312/32743 -11	418/56886 15373/41931 0	13421/0 2144/11277 1	0/14490 5181/9309 -56	1088/12390 1320/12158 2
LVT/RVT Setup WNS (ps) Setup TNS (ns)	6844/6961 -3 -0.003	11385/3596 -24 -1.740	4013/10109 3 0.000	7677/10446 0 0.000	0/22206 14688/7518 -8 -0.091	842/20940 5899/15883 -0 -0.000	52401/0 18120/34281 0 0.000	25312/32743 -11 -0.241	418/56886 15373/41931 0 0.000	13421/0 2144/11277 1 0.000	0/14490 5181/9309 -56 -5.163	1088/12390 1320/12158 2 0.000
LVT/RVT Setup WNS (ps) Setup TNS (ns) #Hold violations	6844/6961 -3 -0.003 0	11385/3596 -24 -1.740 0	4013/10109 3 0.000 0	7677/10446 0 0.000 0	0/22206 14688/7518 -8 -0.091 0	842/20940 5899/15883 -0 -0.000 0	52401/0 18120/34281 0 0.000 0	25312/32743 -11 -0.241 0	418/56886 15373/41931 0 0.000 0	13421/0 2144/11277 1 0.000 0	0/14490 5181/9309 -56 -5.163 0	1088/12390 1320/12158 2 0.000 0
LVT/RVT Setup WNS (ps) Setup TNS (ns) #Hold violations Area (µm <sup>2</sup> )	6844/6961 -3 -0.003 0 12202	11385/3596 -24 -1.740 0 11901	4013/10109 3 0.000 0 8334	7677/10446 0 0.000 0 19158	0/22206 14688/7518 -8 -0.091 0 15040	842/20940 5899/15883 -0 -0.000 0 13767	52401/0 18120/34281 0 0.000 0 57128	25312/32743 -11 -0.241 0 44073	418/56886 15373/41931 0 0.000 0 40678	13421/0 2144/11277 1 0.000 0 19717	0/14490 5181/9309 -56 -5.163 0 14635	1088/12390 1320/12158 2 0.000 0 13127
LVT/RVT Setup WNS (ps) Setup TNS (ns) #Hold violations Area (µm <sup>2</sup> ) Utilization	6844/6961 -3 -0.003 0 12202 66%	11385/3596 -24 -1.740 0 11901 91%	4013/10109 3 0.000 0 8334 58%	7677/10446 0 0.000 0 19158 65%	0/22206 14688/7518 -8 -0.091 0 15040 66%	842/20940 5899/15883 -0 -0.000 0 13767 61%	52401/0 18120/34281 0 0.000 0 57128 65%	25312/32743 -11 -0.241 0 44073 69%	418/56886 15373/41931 0 0.000 0 40678 63%	13421/0 2144/11277 1 0.000 0 19717 62%	0/14490 5181/9309 -56 -5.163 0 14635 70%	1088/12390 1320/12158 2 0.000 0 13127 61%
LVT/RVT Setup WNS (ps) Setup TNS (ns) #Hold violations Area (µm <sup>2</sup> ) Utilization WL (mm)	6844/6961 -3 -0.003 0 12202 66% 157	11385/3596 -24 -1.740 0 11901 91% 137	4013/10109 3 0.000 0 8334 58% 131	7677/10446 0 0.000 0 19158 65% 171	0/22206 14688/7518 -8 -0.091 0 15040 66% 154	842/20940 5899/15883 -0 -0.000 0 13767 61% 156	52401/0 18120/34281 0 0.000 0 57128 65% 539	25312/32743 -11 -0.241 0 44073 69% 425	418/56886 15373/41931 0 0.000 0 40678 63% 423	13421/0 2144/11277 1 0.000 0 19717 62% 149	0/14490 5181/9309 -56 -5.163 0 14635 70% 143	1088/12390 1320/12158 2 0.000 0 13127 61% 135
LVT/RVT Setup WNS (ps) Setup TNS (ns) #Hold violations Area (µm <sup>2</sup> ) Utilization	6844/6961 -3 -0.003 0 12202 66%	11385/3596 -24 -1.740 0 11901 91% 137 0.290	4013/10109 3 0.000 0 8334 58%	7677/10446 0 0.000 0 19158 65%	0/22206 14688/7518 -8 -0.091 0 15040 66%	842/20940 5899/15883 -0 -0.000 0 13767 61%	52401/0 18120/34281 0 0.000 0 57128 65%	25312/32743 -11 -0.241 0 44073 69%	418/56886 15373/41931 0 0.000 0 40678 63%	13421/0 2144/11277 1 0.000 0 19717 62%	0/14490 5181/9309 -56 -5.163 0 14635 70%	1088/12390 1320/12158 2 0.000 0 13127 61%
LVT/RVT Setup WNS (ps) Setup TNS (ns) #Hold violations Area (µm <sup>2</sup> ) Utilization WL (mm)	6844/6961 -3 -0.003 0 12202 66% 157	11385/3596 -24 -1.740 0 11901 91% 137	4013/10109 3 0.000 0 8334 58% 131	7677/10446 0 0.000 0 19158 65% 171	0/22206 14688/7518 -8 -0.091 0 15040 66% 154	842/20940 5899/15883 -0 -0.000 0 13767 61% 156	52401/0 18120/34281 0 0.000 0 57128 65% 539	25312/32743 -11 -0.241 0 44073 69% 425	418/56886 15373/41931 0 0.000 0 40678 63% 423	13421/0 2144/11277 1 0.000 0 19717 62% 149	0/14490 5181/9309 -56 -5.163 0 14635 70% 143	1088/12390 1320/12158 2 0.000 0 13127 61% 135
LVT/RVT           Setup WNS (ps)           Setup TNS (ns)           #Hold violations           Area (μm²)           Utilization           WL (mm)           Leakage (mW)	6844/6961 -3 -0.003 0 12202 66% 157 0.275	11385/3596 -24 -1.740 0 11901 91% 137 0.290	4013/10109 3 0.000 0 8334 58% 131 0.096	7677/10446 0 0.000 0 19158 65% 171 0.253	0/22206 14688/7518 -8 -0.091 0 15040 66% 154 0.243	842/20940 5899/15883 -0 -0.000 0 13767 61% 156 0.104	52401/0 18120/34281 0 0.000 0 57128 65% 539 0.915	25312/32743 -11 -0.241 0 44073 69% 425 0.679	418/56886 15373/41931 0 0.000 0 40678 63% 423 0.423	13421/0 2144/11277 1 0.000 0 19717 62% 149 0.141	0/14490 5181/9309 -56 -5.163 0 14635 70% 143 0.147	1088/12390 1320/12158 2 0.000 0 13127 61% 135 0.063
$\begin{tabular}{ c c c c c } \hline LVT/RVT \\ \hline Setup WNS (ps) \\ \hline Setup TNS (ns) \\ \hline #Hold violations \\ Area (\mu m^2) \\ \hline Utilization \\ \hline WL (mm) \\ \hline Leakage (mW) \\ \hline Total power (mW) \\ \hline Clock area (\mu m^2) \\ \hline \end{tabular}$	6844/6961 -3 -0.003 0 12202 66% 157 0.275 24.0 19	11385/3596 -24 -1.740 0 11901 91% 137 0.290 22.7 10	4013/10109 3 0.000 0 8334 58% 131 0.096 16.3 12	7677/10446 0 0.000 19158 65% 171 0.253 46.1 47	0/22206 14688/7518 -8 -0.091 0 15040 66% 154 0.243 39.1 29	842/20940 5899/15883 -0 -0.000 0 13767 61% 156 0.104 34.9 40	52401/0 18120/34281 0 0.000 0 57128 65% 539 0.915 54.5 100	25312/32743 -11 -0.241 0 44073 69% 425 0.679 48.1 106	418/56886 15373/41931 0 0.000 0 40678 63% 423 0.423 0.423 44.1 95	13421/0 2144/11277 1 0.000 0 19717 62% 149 0.141 28.5 82	0/14490 5181/9309 -56 -5.163 0 14635 70% 143 0.147 23.3 58	1088/12390 1320/12158 2 0.000 0 13127 61% 135 0.063 22.9 51
LVT/RVT Setup WNS (ps) Setup TNS (ns) #Hold violations Area (µm <sup>2</sup> ) Utilization WL (mm) Leakage (mW) Total power (mW) Clock area (µm <sup>2</sup> ) Clock WL (µm)	6844/6961 -3 -0.003 0 12202 66% 157 0.275 24.0 19 1960	11385/3596 -24 -1.740 0 11901 91% 137 0.290 22.7 10 1683	4013/10109 3 0.000 0 8334 58% 131 0.096 16.3 12 1959	7677/10446 0 0.000 0 19158 65% 171 0.253 46.1 47 7605	0/22206 14688/7518 -8 -0.091 0 15040 66% 154 0.243 39.1 29 5933	842/20940 5899/15883 -0 -0.000 0 13767 61% 156 0.104 34.9 40 6326	52401/0 18120/34281 0 0 57128 65% 539 0.915 54.5 100 20133	25312/32743 -11 -0.241 0 44073 69% 425 0.679 48.1 106 18441	418/56886 15373/41931 0 0.0000 40678 63% 423 0.423 44.1 95 16055	13421/0 2144/11277 1 0.000 0 19717 62% 149 0.141 28.5 82 11709	0/14490 5181/9309 -56 -5.163 0 14635 70% 143 0.147 23.3 58 10027	1088/12390 1320/12158 2 0.000 0 13127 61% 135 0.063 22.9 51 9734
LVT/RVT Setup WNS (ps) Setup TNS (ns) #Hold violations Area (µm <sup>2</sup> ) Utilization WL (mm) Leakage (mW) Total power (mW) Clock area (µm <sup>2</sup> ) Clock WL (µm) Runtime (min)	6844/6961 -3 -0.003 0 12202 66% 157 0.275 24.0 19 1960 49	11385/3596 -24 -1.740 0 11901 91% 137 0.290 22.7 10 1683 73	4013/10109 3 0.000 0 8334 58% 131 0.096 16.3 12 1959 113	7677/10446 0 0.000 19158 65% 171 0.253 46.1 47	0/22206 14688/7518 -8 -0.091 0 15040 66% 154 0.243 39.1 29 5933 111	842/20940 5899/15883 -0 -0.000 0 13767 61% 156 0.104 34.9 40 6326 93	52401/0 18120/34281 0 0 57128 65% 539 0.915 54.5 100 20133 189	25312/32743 -11 -0.241 0 44073 69% 425 0.679 48.1 106 18441 244	418/56886 15373/41931 0 0.000 0 40678 63% 423 0.423 0.423 44.1 95	13421/0 2144/11277 1 0.000 0 19717 62% 149 0.141 28.5 82 11709 59	0/14490 5181/9309 -56 -5.163 0 14635 70% 143 0.147 23.3 58 10027 91	1088/12390 1320/12158 2 0.000 0 13127 61% 135 0.063 22.9 51 9734 63
LVT/RVT Setup WNS (ps) Setup TNS (ns) #Hold violations Area (µm <sup>2</sup> ) Utilization WL (mm) Leakage (mW) Total power (mW) Clock area (µm <sup>2</sup> ) Clock WL (µm) Runtime (min) Design (clk period)	6844/6961 -3 -0.003 0 12202 66% 157 0.275 24.0 19 19 600 49	11385/3596 -24 -1.740 0 11901 91% 137 0.290 22.7 10 1683 73 AES (800ps)	4013/10109 3 0.000 0 8334 58% 131 0.096 16.3 12 1959 113	7677/10446 0 0.0000 0 19158 65% 171 0.253 46.1 47 7605 97	0/22206 14688/7518 -8 -0.091 0 15040 66% 154 0.243 39.1 29 5933 111 DES (750ps	842/20940 5899/15883 -0 0 0 13767 61% 156 0.104 34.9 40 6326 93	52401/0 18120/34281 0 0.0000 57128 65% 539 0.915 54.5 100 20133 189	25312/32743 -11 -0.241 0 44073 69% 425 0.679 48.1 106 18441 244 JPEG (900ps)	418/56886 15373/41931 0 0.0000 0 40678 63% 423 0.423 44.1 95 16055 579	13421/0 2144/11277 1 0.0000 0 197117 62% 149 0.141 28.5 82 11709 59	0/14490 5181/9309 -56 -5.163 0 14635 70% 143 0.147 23.3 58 10027 91 MPEG (700ps	1088/12390 1320/12158 2 0.0000 0 13127 61% 135 0.063 22.9 51 9734 63
$\label{eq:linear_states} \begin{split} & LVT/RVT \\ \hline Setup WNS (ps) \\ Setup TNS (ns) \\ \hline Htold violations \\ Area (\mu m^2) \\ \hline Utilization \\ WL (mm) \\ Leakage (mW) \\ \hline Total power (mW) \\ \hline Clock area (\mu m^2) \\ \hline Clock WL (\mu m) \\ \hline Runtime (min) \\ \hline Design (clk period) \\ \#Instances \end{split}$	6844/6961 -3 -0.003 0 12202 66% 157 0.275 24.0 19 1960 49 12883	11385/3596 -24 -1.740 0 11901 91% 137 0.290 22.7 10 1683 73 AES (800ps) 14480	4013/10109 3 0.000 0 8334 58% 131 0.096 16.3 12 1959 113 14043	7677/10446 0 0.000 19158 65% 171 0.253 46.1 47 7605 97 17636	0/22206 14688/7518 -8 -0.091 0 15040 66% 154 0.243 39.1 29 5933 111 DES (750ps 20178	842/20940 5899/15883 -0 -0.000 0 13767 61% 156 0.104 34.9 40 6326 93 ) 19788	52401/0 18120/34281 0 0.000 0 57128 65% 539 0.915 54.5 100 20133 189 51991	25312/32743 -11 -0.241 0 44073 69% 425 0.679 48.1 106 18441 244 JPEG (900ps) 57388	418/56886 15373/41931 0 0.000 0 40678 63% 423 0.423 44.1 95 16055 579	13421/0 2144/11277 1 0.000 0 19717 62% 149 0.141 28.5 82 11709 59 59 11977	0/14490 5181/9309 -56 -5.163 0 14635 70% 143 0.147 23.3 58 10027 91 MPEG (700ps 14514	1088/12390 1320/12158 2 0.000 0 13127 61% 135 0.063 22.9 51 9734 63 5) 13445
$\label{eq:linear_state} \begin{split} & LVT/RVT \\ \hline Setup WNS (ps) \\ Setup TNS (ns) \\ \hline & HIold violations \\ Area (\mu m^2) \\ \hline & Utilization \\ WL (mm) \\ \hline & Leakage (mW) \\ \hline & Total power (mW) \\ \hline & Clock area (\mu m^2) \\ \hline & Clock WL (\mu m) \\ \hline & Runtime (min) \\ \hline & Design (clk period) \\ \hline & HInstances \\ 12T/8T \end{split}$	6844/6961 -3 -0.003 0 12202 66% 157 0.275 24.0 19 1960 49 12883 12883	11385/3596 -24 -1.740 0 11901 91% 137 0.290 22.7 10 1683 73 AES (800ps) 14480 0/14480	4013/10109 3 0.000 0 8334 58% 131 0.096 16.3 12 1959 113 14043 0/14043	7677/10446 0 0.000 0 19158 65% 171 0.253 46.1 47 7605 97 	0/22206 14688/7518 -8 -0.091 0 15040 66% 154 0.243 39.1 29 5933 111 DES (750ps 20178 0/20178	842/20940 5899/15883 -0 -0.000 0 13767 61% 156 0.104 34.9 40 6326 93 ) 19788 1427/18361	52401/0 18120/34281 0 0.0000 0 57128 65% 539 0.915 54.5 100 20133 189 51991 51991/0	25312/32743 -11 -0.241 0 44073 69% 425 0.679 48.1 106 18441 244 JPEG (900ps) 57388 0/57388	418/56886 15373/41931 0 0.000 40678 63% 423 0.423 44.1 95 16055 579 55287 212/56075	13421/0 2144/11277 1 0.000 0 19717 62% 149 0.141 28.5 82 11709 59 59 N 11977 11977/0	0/14490 5181/9309 -56 -5.163 0 14635 70% 143 0.147 23.3 58 10027 91 MPEG (700ps 14514 0/14514	1088/12390 1320/12158 2 0.0000 0 13127 61% 135 0.063 22.9 51 9734 63 5) 13445 253/13192
LVT/RVT           Setup WNS (ps)           Setup TNS (ns)           #Hold violations           Area (μm²)           Utilization           WL (mm)           Leakage (mW)           Total power (mW)           Clock area (μm²)           Clock area (μm²)           Clock WL (μm)           Runtime (min)           Design (clk period)           #Instances           127/8T           LVT/RVT	6844/6961 -3 -0.003 0 12202 66% 157 0.275 24.0 19 1960 49 12883 12883 12883(0 5453/7430	11385/3596 -24 -1.740 0 11901 91% 137 0.290 22.7 10 1683 73 AES (800ps) 14480 0/14480 9557/4923	4013/10109 3 0.000 0 8334 58% 131 0.096 16.3 12 1959 113 14043 0/14043 4907/9136	7677/10446 0 0.000 0 19158 65% 171 0.253 46.1 47 7605 97 17636 17636 17636/0 6754/10882	0/22206 14688/7518 -8 -0.091 0 15040 66% 154 0.243 39.1 29 5933 111 DES (750ps 20178 0/20178 12355/7823	842/20940 5899/15883 -0 -0.000 0 13767 61% 156 0.104 34.9 40 6326 93 ) 19788	52401/0 18120/34281 0 0.000 0 57128 65% 539 0.915 54.5 100 20133 189 51991 51991/0 14054/37937	25312/32743 -11 -0.241 0 44073 69% 425 0.679 48.1 106 18441 244 JPEG (900ps) 57388 0/57388 25056/32332	418/56886 15373/41931 0 0.000 0 40678 63% 423 0.423 44.1 95 16055 579 56287 212/56075 14807/41480	13421/0 2144/11277 1 0.000 0 19717 62% 149 0.141 28.5 82 11709 59 59 11977	0/14490 5181/9309 -56 -5.163 0 14635 70% 143 0.147 23.3 58 10027 91 MPEG (700ps 14514 0/14514 4144/10370	1088/12390 1320/12158 2 0.000 0 13127 61% 135 0.063 22.9 51 9734 63 ) 13445 253/13192 1079/12366
LVT/RVT Setup WNS (ps) Setup TNS (ns) #Hold violations Area (µm <sup>2</sup> ) Utilization WL (mm) Leakage (mW) Total power (mW) Clock area (µm <sup>2</sup> ) Clock WL (µm) Runtime (min) Design (clk period) #Instances 12T/8T LVT/RVT Setup WNS (ps)	6844/6961 -3 -0.003 0 12202 66% 157 0.275 24.0 19 19 1960 49 12883 12883/0 5453/7430 0	11385/3596 -24 -1.740 0 11901 91% 137 0.290 22.7 10 1683 73 AES (800ps) 14480 0/14480 9557/4923 -11	4013/10109 3 0.000 0 8334 58% 131 0.096 16.3 12 1959 113 14043 0/14043 4907/9136 2	7677/10446 0 0.000 0 19158 65% 171 0.253 46.1 47 7605 97 17636 17636(0 6754/10882 -0	0/22206 14688/7518 -8 -0.091 0 15040 66% 154 0.243 39.1 29 5933 111 DES (750ps 20178 0/20178 0/20178 0/20178 3 -6	842/20940 5899/15883 -0 -0.000 0 13767 61% 156 0.104 34.9 40 6326 93 ) 19788 1427/18361 5087/14701 1	52401/0 18120/34281 0 0.0000 0 57128 65% 539 0.915 54.5 100 20133 189 51991 51991/0 51991/0 14054/37937 0	25312/32743 -11 -0.241 0 44073 69% 425 0.679 48.1 106 18441 244 JPEG (900ps) 57388 0/57388 0/57388 25056/32332 -11	418/56886 15373/41931 0 0.0000 0 40678 63% 423 0.423 0.423 44.1 95 16055 579 56287 212/56075 212/56075 14807/41480 0	13421/0 2144/11277 1 0.0000 0 197117 62% 149 0.141 28.5 82 11709 59 N 11977 119770 174/10203 1	0/14490 5181/9309 -56 0 14635 70% 143 0.147 23.3 58 10027 91 MPEG (700ps 14514 0/14514 0/14514 144/10370 -12	1088/12390 1320/12158 2 0.0000 0 13127 61% 135 0.063 22.9 51 9734 63 3) 13445 253/13192 1079/12366 3
$eq:linear_line$	6844/6961 -3 -0.003 0 12202 66% 157 0.275 24.0 19 1960 49 12883 128830 5453/7430 0 0.000	11385/3596 -24 -1.740 0 11901 91% 137 0.290 22.7 10 1683 73 AES (800ps) 14480 0/14480 9557/4923 -11 -0.280	4013/10109 3 0.000 0 8334 58% 131 0.096 16.3 12 1959 113 14043 0/14043 4907/9136 2 0.000	7677/10446 0 0.000 0 19158 65% 171 0.253 46.1 47 7605 97 17636 176360 176360 6754/10882 -0 -0.000	0/22206 14688/7518 -8 -0.091 0 15040 66% 0.243 39.1 29 5933 111 DES (750ps 20178 0/20178 0/20178 12355/7823 -6 -0.024	842/20940 5899/15883 -0 -0.000 0 13767 61% 156 0.104 34.9 40 6326 93 ) 19788 1427/18361 5087/14701 1 0.000	52401/0 18120/34281 0 0.000 0 57128 65% 539 0.915 54.5 100 20133 189 51991/0 51991/0 14054/37937 0 0	25312/32743 -11 -0.241 0 44073 69% 425 0.679 48.1 106 18441 244 JPEG (900ps) 57388 0/57388 25056/32332 -11 -0.375	418/56886 15373/41931 0 0.000 0 40678 63% 423 0.423 44.1 95 16055 579 56287 212/56075 14807/41480 0 0	13421/0 2144/11277 1 0.000 0 19717 62% 149 0.141 28.5 82 11709 59 59 N 11977 11977/0 1774/10203 1 0.000	0/14490 5181/9309 -55 -55.163 0 14635 70% 143 0.147 23.3 58 10027 91 MPEG (700ps 14514 0/14514 4144/10370 -12 -0.455	1088/12390 1320/12158 2 0.0000 0 13127 61% 135 0.063 22.9 51 9734 63 5) 13445 253/13192 1079/12366 3 0.000
LVT/RVT Setup WNS (ps) Setup TNS (ns) #Hold violations Area (µm <sup>2</sup> ) Utilization WL (mm) Leakage (mW) Total power (mW) Clock area (µm <sup>2</sup> ) Clock WL (µm) Runtime (min) Design (clk period) #Instances 12T/8T LVT/RVT Setup WNS (ps)	6844/6961 -3 -0.003 0 12202 66% 157 0.275 24.0 19 19 1960 49 12883 12883/0 5453/7430 0	11385/3596 -24 -1.740 0 11901 91% 137 0.290 22.7 10 1683 73 AES (800ps) 14480 9557/4923 -11 -0.280 0	4013/10109 3 0.000 0 8334 58% 131 0.096 16.3 12 1959 113 14043 0/14043 4907/9136 2	7677/10446 0 0.000 0 19158 65% 171 0.253 46.1 47 7605 97 17636 17636(0 6754/10882 -0	0/22206 14688/7518 -8 -0.091 0 15040 66% 154 0.243 39.1 29 5933 111 DES (750ps 20178 0/20178 0/20178 0/20178 3 -6	842/20940 5899/15883 -0 -0.000 0 13767 61% 156 0.104 34.9 40 6326 93 ) 19788 1427/18361 5087/14701 1	52401/0 18120/34281 0 0.0000 0 57128 65% 539 0.915 54.5 100 20133 189 51991 51991/0 51991/0 14054/37937 0	25312/32743 -11 -0.241 0 44073 69% 425 0.679 48.1 106 18441 244 JPEG (900ps) 57388 0/57388 0/57388 25056/32332 -11	418/56886 15373/41931 0 0.0000 0 40678 63% 423 0.423 0.423 44.1 95 16055 579 56287 212/56075 212/56075 14807/41480 0	13421/0 2144/11277 1 0.0000 0 197117 62% 149 0.141 28.5 82 11709 59 N 11977 119770 174/10203 1	0/14490 5181/9309 -56 0 14635 70% 143 0.147 23.3 58 10027 91 MPEG (700ps 14514 0/14514 0/14514 144/10370 -12	1088/12390 1320/12158 2 0.0000 0 13127 61% 135 0.063 22.9 51 9734 63 3) 13445 253/13192 1079/12366 3
$eq:linear_line$	6844/6961 -3 -0.003 0 12202 66% 157 0.275 24.0 19 1960 49 12883 128830 5453/7430 0 0.000	11385/3596 -24 -1.740 0 11901 91% 137 0.290 22.7 10 1683 73 AES (800ps) 14480 0/14480 9557/4923 -11 -0.280	4013/10109 3 0.000 0 8334 58% 131 0.096 16.3 12 1959 113 14043 0/14043 4907/9136 2 0.000	7677/10446 0 0.000 0 19158 65% 171 0.253 46.1 47 7605 97 17636 176360 176360 6754/10882 -0 -0.000	0/22206 14688/7518 -8 -0.091 0 15040 66% 0.243 39.1 29 5933 111 DES (750ps 20178 0/20178 0/20178 12355/7823 -6 -0.024	842/20940 5899/15883 -0 -0.000 0 13767 61% 156 0.104 34.9 40 6326 93 ) 19788 1427/18361 5087/14701 1 0.000	52401/0 18120/34281 0 0.000 0 57128 65% 539 0.915 54.5 100 20133 189 51991/0 51991/0 14054/37937 0 0	25312/32743 -11 -0.241 0 44073 69% 425 0.679 48.1 106 18441 244 JPEG (900ps) 57388 0/57388 25056/32332 -11 -0.375	418/56886 15373/41931 0 0.000 0 40678 63% 423 0.423 44.1 95 16055 579 56287 212/56075 14807/41480 0 0	13421/0 2144/11277 1 0.000 0 19717 62% 149 0.141 28.5 82 11709 59 59 N 11977 11977/0 1774/10203 1 0.000	0/14490 5181/9309 -55 -55.163 0 14635 70% 143 0.147 23.3 58 10027 91 MPEG (700ps 14514 0/14514 4144/10370 -12 -0.455	1088/12390 1320/12158 2 0.0000 0 13127 61% 135 0.063 22.9 51 9734 63 5) 13445 253/13192 1079/12366 3 0.000
$\label{eq:linear_states} \begin{split} & \text{LVT/RVT} \\ \hline \text{Setup WNS (ps)} \\ \hline \text{Setup TNS (ns)} \\ \hline \text{#Hold violations} \\ \hline \text{Area } (\mu m^2) \\ \hline \text{Utilization} \\ \hline \text{WL (mm)} \\ \hline \text{Leakage (mW)} \\ \hline \text{Total power (mW)} \\ \hline \text{Clock area } (\mu m^2) \\ \hline \text{Clock WL } (\mu m) \\ \hline \text{Runtime (min)} \\ \hline \hline \text{Design (clk period)} \\ \hline \text{#Instances} \\ 12T/8T \\ \hline \text{LVT/RVT} \\ \hline \text{Setup WNS (ps)} \\ \hline \text{Setup TNS (ns)} \\ \hline \text{#Hold violations} \\ \hline \end{split}$	6844/6961 -3 -0.003 0 12202 66% 157 0.275 24.0 19 1960 49 12883 12883/0 5453/7430 0 0.000 0.000	11385/3596 -24 -1.740 0 11901 91% 137 0.290 22.7 10 1683 73 AES (800ps) 14480 9557/4923 -11 -0.280 0	4013/10109 3 0.000 0 8334 58% 131 0.096 16.3 12 1959 113 14043 0/14043 4907/9136 2 0.000 0	7677/10446 0 0.000 19158 65% 171 0.253 46.1 47 7605 97 17636 17636 17636/0 6754/10882 -0 -0.000 0	0/22206 14688/7518 -8 -0.091 0 15040 66% 154 0.243 39.1 29 5933 39.1 29 5933 111 DES (750ps 20178 0/20178 12355/7823 -6 -0.024 0	842/20940 5899/15883 -0 -0.000 0 13767 61% 156 0.104 34.9 40 6326 93 93 ) 19788 1427/18361 5087/14701 1 1 0.000 0	52401/0 18120/34281 0 0.000 0 57128 65% 539 0.915 54.5 100 20133 189 51991 51991/0 14054/37937 0 0.000 0	25312/32743 -11 -0.241 0 44073 69% 425 0.679 48.1 106 18441 244 JPEG (900ps) 57388 25056/32332 -11 -0.375 0	418/56886 15373/41931 0 0.000 0 40678 63% 423 0.423 44.1 95 16055 579 56287 212/56075 14807/41480 0 0.000 0	13421/0 2144/11277 1 0.000 0 19717 62% 149 0.141 28.5 82 11709 59 59 59 N 11977 11977/0 1774/10203 1 0.000 0	0/14490 5181/9309 -56 -5.163 0 14635 70% 143 0.147 23.3 58 10027 91 MPEG (700ps 14514 0/14514 4144/10370 -12 -0.455 0	1088/12390 1320/12158 2 0.000 0 13127 61% 135 0.063 22.9 51 9734 63 3 13445 253/13192 1079/12366 3 0.000 0
LVT/RVT Setup WNS (ps) Setup TNS (ns) #Hold violations Area (µm <sup>2</sup> ) Utilization WL (mm) Leakage (mW) Total power (mW) Clock area (µm <sup>2</sup> ) Clock WL (µm) Runtime (min) Design (clk period) #Instances 12T/8T LVT/RVT Setup WNS (ps) Setup TNS (ns) #Hold violations Area (µm <sup>2</sup> ) Utilization	6844/6961 -3 -0.003 0 12202 66% 157 0.275 24.0 19 1960 49 12883 12883 12883 12883 0 5453/7430 0 0 0 0 0 0 11294 63%	11385/3596 -24 -1.740 0 11901 91% 137 0.290 22.7 10 1683 73 AES (800ps) 14480 0/14480 9557/4923 -11 -0.280 0 10244 74%	4013/10109 3 0.000 0 8334 58% 131 0.096 16.3 12 1959 113 14043 0/14043 4907/9136 2 0.000 0 7402 54%	7677/10446 0 0.000 0 19158 65% 171 0.253 46.1 47 7605 97 17636 17636/0 6754/10882 -0 -0.000 0 18637 65%	0/22206 14688/7518 -8 -0.091 0 15040 66% 154 0.243 39.1 29 5933 111 DES (750ps 20178 0/20178 12355/7823 -6 -0.024 0 14359 64%	842/20940 5899/15883 -0 000 0 13767 61% 156 0.104 34.9 40 6326 93 19788 1427/18361 5087/14701 1 0.000 0 13447 61%	52401/0 18120/34281 0 0.0000 0 57128 65% 539 0.915 54.5 100 20133 189 51991 51991/0 14054/37937 0 0.000 0 0 54647 60%	25312/32743 -11 -0.241 0 44073 69% 425 0.679 48.1 106 18441 244 JPEG (900ps) 57388 0/57388 25056/32332 -11 -0.375 0 0 44758 71%	418/56886 15373/41931 0 0.0000 0 40678 63% 423 0.423 0.423 44.1 95 16055 56287 212/56075 14807/4180 0 0.000 0 0 40400 64%	13421/0 2144/11277 1 0.0000 0 197117 62% 149 0.141 28.5 82 11709 59 N 11977 11977/0 1774/10203 1 0.0000 0 19434 63%	0/14490 5181/9309 -56 -5.163 0 14635 70% 143 0.147 23.3 58 10027 91 MPEG (700ps 14514 0/14514 0/14514 0/14514 0-12 -0.455 0 14038 67%	1088/12390 1320/12158 2 0.0000 0 13127 61% 135 0.063 22.9 51 9734 63 3 13445 253/13192 1079/12366 3 0.000 0 12794 60%
$eq:linear_line$	6844/6961 -3 -0.003 0 12202 66% 157 0.275 24.0 19 1960 49 1960 49 12883 128830 5453/7430 0 0.000 0 0.000 0 0 11294 155	11385/3596 -24 -1.740 0 11901 91% 137 0.290 22.7 10 1683 73 AES (800ps) 14480 0/14480 0/14480 0/14480 0557/4923 -11 -0.280 0 10 10244 74% 133	4013/10109 3 0.000 0 8334 58% 131 0.096 16.3 12 1959 113 14043 0/14043 0/14043 4907/9136 2 0.000 0 7402 54% 128	$\begin{array}{r} 7677/10446\\ 0\\ 0.000\\ 0\\ 19158\\ 65\%\\ 171\\ 0.253\\ 46.1\\ 47\\ 7605\\ 97\\ \hline \\ 176360\\ 176360\\ 176360\\ 176360\\ 176360\\ 176360\\ 0\\ 6754/10882\\ -0\\ 0\\ 18637\\ 65\%\\ 165\\ \end{array}$	0/22206 14688/7518 -8 -0.091 0 15040 66% 0.243 39.1 29 5933 1111 DES (750ps 20178 0/20	842/20940 5899/15883 -0 -0.000 0 13767 61% 156 0.104 34.9 40 6326 93 ) 19788 1427/18361 5087/14701 1 0.0000 0 13447 61% 153	52401/0 18120/34281 0 0.0000 0 57128 65% 539 0.915 54.5 100 20133 189 51991/0 51991/0 51991/0 51991/0 14054/37937 0 0 0.0000 0 54647 60% 591	25312/32743 -11 -0.241 0 44073 69% 425 0.679 48.1 106 18441 244 JPEC (900ps) 57388 0/57388 0/57388 25056/32332 -11 -0.375 0 44758 71% 483	418/56886 15373/41931 0 0.000 0 40678 63% 423 0.423 44.1 95 16055 579 56287 212/56075 212/56075 14807/41480 0 0 0.000 0 0 40400 64% 479	13421/0 2144/11277 1 0.000 0 19717 62% 149 0.141 28.5 82 11709 59 59 N 11977 11977/0 1774/10203 174/10203 1 0.000 0 0 19434 63% 159	0/14490 5181/9309 -56 -5.163 0 14635 70% 143 0.147 23.3 58 10027 91 MPEG (700ps 14514 0/14514 0/14514 0/14514 0/14514 0/14515 0 -12 -0.455 0 14038 67% 138	1088/12390 1320/12158 2 0.0000 0 13127 61% 135 0.063 22.9 51 9734 63 5) 13445 253/13192 1079/12366 3 0.000 0 12794 60% 140
$eq:linear_line$	6844/6961 -3 -0.003 0 12202 66% 157 0.275 24.0 19 1960 49 12883 12883/0 5453/7430 0 0 0.000 0 0 11294 63% 155 0.220	11385/3596 -24 -1.740 0 11901 91% 137 0.290 22.7 10 1683 73 AES (800ps) 14480 0/14480 9557/4923 -11 -0.280 0 10244 74% 133 0.224	4013/10109 3 0.000 0 8334 58% 131 0.096 16.3 12 1959 113 14043 0/14043 4907/9136 2 0.000 0 7402 54% 128 0.078	$\begin{array}{r} 7677/10446\\ 0\\ 0.000\\ 0\\ 19158\\ 65\%\\ 171\\ 0.253\\ 46.1\\ 47\\ 7605\\ 97\\ \hline \\ 17636\\ 17636\\ 17636/0\\ 6754/10882\\ -0\\ 6754/10882\\ -0\\ 18637\\ 65\%\\ 165\\ 0.220\\ \end{array}$	0/22206 14688/7518 -8 -0.091 0 15040 66% 154 0.243 39.1 29 5933 111 DES (750ps 20178 12355/7823 -6 -0.024 0 14359 64% 150 0.207	842/20940 5899/15883 -0 -0.000 0 13767 61% 156 0.104 34.9 40 6326 93 ) 19788 1427/18361 5087/14701 1 0.000 0 13447 61% 153 0.094	52401/0 18120/34281 0 0.000 0 57128 65% 539 0.915 54.5 100 20133 189 51991 51991/0 14054/37937 0 0.0000 0 0 54647 60% 591 0.711	25312/32743 -11 -0.241 0 44073 69% 425 0.679 48.1 106 18441 244 1PEG (900ps) 57388 0/57388 25056/32332 -11 -11 0 44758 71% 483 0.699	418/56886 15373/41931 0 0.000 0 40678 63% 423 0.423 44.1 95 16055 579 56287 212/56075 14807/41480 0 0.000 0 0 40400 64% 479 0.413	13421/0 2144/11277 1 0.000 0 19717 62% 149 0.141 28.5 82 11709 59 59 N 11977 11977/0 1774/10203 1 0.000 0 0 19434 63% 159 0.129	0/14490 5181/9309 -56 -5.163 0 14635 70% 143 0.147 23.3 58 10027 91 MPEG (700ps 14514 0/14514 4144/10370 -12 -0.455 0 14038 67% 138 0.116	1088/12390 1320/12158 2 0.000 0 13127 61% 135 0.063 22.9 51 9734 63 5) 13445 253/13192 1079/12366 3 0.000 0 12794 60% 140 0.054
$\label{eq:linear_state} \begin{split} & \text{LVT/RVT} \\ \hline \text{Setup WNS (ps)} \\ \hline \text{Setup TNS (ns)} \\ \hline \text{Hold violations} \\ \hline \text{Area } (\mu m^2) \\ \hline \text{Utilization} \\ \hline \text{WL (mm)} \\ \hline \text{Leakage (mW)} \\ \hline \text{Leakage (mW)} \\ \hline \text{Clock area } (\mu m^2) \\ \hline \text{Clock area } (\mu m^2) \\ \hline \text{Clock area } (\mu m^2) \\ \hline \text{Clock WL } (\mu m) \\ \hline \text{Runtime (min)} \\ \hline \hline \text{Design (clk period)} \\ \hline \text{#Instances} \\ 12T/8T \\ \hline \text{LVT/RVT} \\ \hline \text{Setup WNS (ps)} \\ \hline \text{Setup TNS (ns)} \\ \hline \text{#Hold violations} \\ \hline \text{Area } (\mu m^2) \\ \hline \text{Utilization} \\ \hline \text{WL (mm)} \\ \hline \text{Leakage (mW)} \\ \hline \text{Total power (mW)} \\ \hline \end{split}$	6844/6961 -3 -0.003 0 12202 66% 157 0.275 24.0 19 1960 49 19 1960 49 12883 12883/0 5453/7430 0 0.000 0 0 0 11294 63% 155 0.220 21.2	11385/3596 -24 -1.740 0 11901 91% 137 0.290 22.7 10 1683 73 AES (800ps) 14480 9557/4923 -11 -0.280 0 10244 74% 133 0.224 19.2	4013/10109 3 0.000 0 8334 58% 131 0.096 16.3 12 1959 113 14043 0/14043 4907/9136 2 0.000 0 7402 54% 128 0.078 13.7	7677/10446 0 0.000 0 19158 65% 171 0.253 46.1 47 7605 97 17636 17636/0 6754/10882 -0 -0.000 0 18637 65% 165 0.220 41.8	0/22206 14688/7518 -8 -0.091 0 15040 66% 154 0.243 39.1 29 5933 39.1 29 5933 111 DES (750ps 20178 0/20178 12355/7823 -6 -0.024 0 14359 64% 150 0.207 35.5	842/20940 5899/15883 -0 0 0 13767 61% 156 0.104 34.9 40 6326 93 ) 19788 1427/18361 5087/14701 1 0.0000 0 13447 61% 153 0.094 32.3	52401/0 18120/34281 0 0.000 0 57128 65% 539 0.915 54.5 100 20133 189 51991 51991/0 14054/37937 0 0.000 0 0 54647 60% 591 0.711 52.0	25312/32743 -11 -0.241 0 44073 69% 425 0.679 48.1 106 18441 244 JPEG (900ps) 57388 25056/32332 -11 -0.375 0 44758 71% 483 0.699 42.7	418/56886 15373/41931 0 0.000 0 40678 63% 423 0.423 44.1 95 16055 579 56287 212/56075 14807/41480 0 0.000 0 0 40400 64% 479 0.413 38.4	13421/0 2144/11277 1 0.000 0 19717 62% 149 0.141 28.5 82 11709 59 59 59 59 11977 11977/0 1774/10203 1 0.000 0 0 19434 63% 159 0.129 26.8	0/14490 5181/9309 -56 -5.163 0 14635 70% 143 0.147 23.3 58 10027 91 MPEG (700ps 14514 0/14514 4144/10370 -12 -0.455 0 14038 67% 138 0.116 21.4	1088/12390 1320/12158 2 0.0000 0 13127 61% 135 0.063 22.9 51 9734 63 3 13445 253/13192 1079/12366 3 0.000 0 12794 60% 140 0.054 21.0
$eq:linear_line$	6844/6961 -3 -0.003 0 12202 66% 157 0.275 24.0 19 1960 49 1983 12883 12883 12883/0 5453/7430 0 0 0 0 0 11294 63% 155 0.220 21.2 13	11385/3596 -24 -1.740 0 11901 91% 137 0.290 22.7 10 1683 73 AES (800ps) 14480 0/14480 9557/4923 -11 -0.280 0 10244 74% 133 0.224 19.2 11	4013/10109 3 0.000 0 8334 58% 131 0.096 16.3 12 1959 113 14043 0/14043 4907/9136 2 0.000 0 7402 54% 128 0.078 13.7 11	7677/10446 0 0.000 0 19158 65% 171 0.253 46.1 47 7605 97 17636 17636/0 6754/10882 -0 -0.000 0 18637 65% 165 0.220 41.8 47	0/22206 14688/7518 -8 -0.091 0 15040 66% 154 0.243 39.1 29 5933 111 DES (750ps 20178 20178 20178 20178 20178 12355/7823 -6 -0.024 0 14359 64% 150 0.207 35.5 30	842/20940 5899/15883 -0 -0.000 0 13767 61% 13767 0.104 34.9 40 6326 93 ) 19788 1427/18361 5087/14701 1 0.0000 0 13447 61% 153 0.094 32.3 32	52401/0 18120/34281 0 0.0000 0 57128 65% 539 0.915 54.5 100 20133 189 51991/0 14054/37937 0 0.0000 0 0.0000 0 554647 60% 591 0.711 52.0 111	25312/32743 -11 -0.241 0 44073 69% 425 0.679 48.1 106 18441 244 JPEG (900ps) 57388 0/57388 25056/32332 -11 -0.375 0 44758 71% 483 0.699 42.7 85	418/56886 15373/41931 0 0.0000 0 40678 63% 423 0.423 44.1 95 16055 579 56287 212/56075 14807/4180 0 0.000 0 0 40400 64% 479 0.413 38.4 93	13421/0 2144/11277 1 0.0000 0 19717 62% 149 0.141 28.5 82 11709 59 N 11977 11977/0 1774/10203 1 0.0000 0 0 19434 63% 159 0.129 26.8 101	0/14490 5181/9309 -56 -5.163 0 14635 70% 143 0.147 23.3 58 10027 91 MPEG (700ps 14514 0/14515 0/1455 0/145	1088/12390 1320/12158 2 0.0000 0 13127 61% 135 0.063 22.9 51 9734 63 3 13445 253/13192 1079/12366 3 0.000 0 0 12794 60% 140 0.054 21.0 46
$\label{eq:linear_state} \begin{split} & \text{LVT/RVT} \\ \hline \text{Setup WNS (ps)} \\ \hline \text{Setup TNS (ns)} \\ \hline \text{Hold violations} \\ \hline \text{Area } (\mu m^2) \\ \hline \text{Utilization} \\ \hline \text{WL (mm)} \\ \hline \text{Leakage (mW)} \\ \hline \text{Leakage (mW)} \\ \hline \text{Clock area } (\mu m^2) \\ \hline \text{Clock area } (\mu m^2) \\ \hline \text{Clock area } (\mu m^2) \\ \hline \text{Clock WL } (\mu m) \\ \hline \text{Runtime (min)} \\ \hline \hline \text{Design (clk period)} \\ \hline \text{#Instances} \\ 12T/8T \\ \hline \text{LVT/RVT} \\ \hline \text{Setup WNS (ps)} \\ \hline \text{Setup TNS (ns)} \\ \hline \text{#Hold violations} \\ \hline \text{Area } (\mu m^2) \\ \hline \text{Utilization} \\ \hline \text{WL (mm)} \\ \hline \text{Leakage (mW)} \\ \hline \text{Total power (mW)} \\ \hline \end{split}$	6844/6961 -3 -0.003 0 12202 66% 157 0.275 24.0 19 1960 49 19 1960 49 12883 12883/0 5453/7430 0 0.000 0 0 0 11294 63% 155 0.220 21.2	11385/3596 -24 -1.740 0 11901 91% 137 0.290 22.7 10 1683 73 AES (800ps) 14480 9557/4923 -11 -0.280 0 10244 74% 133 0.224 19.2	4013/10109 3 0.000 0 8334 58% 131 0.096 16.3 12 1959 113 14043 0/14043 4907/9136 2 0.000 0 7402 54% 128 0.078 13.7	7677/10446 0 0.000 0 19158 65% 171 0.253 46.1 47 7605 97 17636 17636/0 6754/10882 -0 -0.000 0 18637 65% 165 0.220 41.8	0/22206 14688/7518 -8 -0.091 0 15040 66% 154 0.243 39.1 29 5933 39.1 29 5933 111 DES (750ps 20178 0/20178 12355/7823 -6 -0.024 0 14359 64% 150 0.207 35.5	842/20940 5899/15883 -0 0 0 13767 61% 156 0.104 34.9 40 6326 93 ) 19788 1427/18361 5087/14701 1 0.0000 0 13447 61% 153 0.094 32.3	52401/0 18120/34281 0 0.000 0 57128 65% 539 0.915 54.5 100 20133 189 51991 51991/0 14054/37937 0 0.000 0 0 54647 60% 591 0.711 52.0	25312/32743 -11 -0.241 0 44073 69% 425 0.679 48.1 106 18441 244 JPEG (900ps) 57388 25056/32332 -11 -0.375 0 44758 71% 483 0.699 42.7	418/56886 15373/41931 0 0.000 0 40678 63% 423 0.423 44.1 95 16055 579 56287 212/56075 14807/41480 0 0.000 0 0 40400 64% 479 0.413 38.4	13421/0 2144/11277 1 0.000 0 19717 62% 149 0.141 28.5 82 11709 59 59 59 59 11977 11977/0 1774/10203 1 0.000 0 0 19434 63% 159 0.129 26.8	0/14490 5181/9309 -56 -5.163 0 14635 70% 143 0.147 23.3 58 10027 91 MPEG (700ps 14514 0/14514 4144/10370 -12 -0.455 0 14038 67% 138 0.116 21.4	1088/12390 1320/12158 2 0.0000 0 13127 61% 135 0.063 22.9 51 9734 63 3 13445 253/13192 1079/12366 3 0.000 0 12794 60% 140 0.054 21.0



Fig. 15. Pareto curves of performance-area tradeoff for implementations with 8 T-only, 12 T-only and mixed cells.

compared to 12 T-only designs, which is mainly due to our wirelength-aware cell mapping and reduced total cell area (i.e., a more compact layout). Furthermore, in our experimental

flow we use command *ccopt\_design* from *Innovus* to perform clock tree synthesis. Results show that our optimized designs with mixed cell heights have similar clock tree metrics



Fig. 16. Iso-performance power comparison with voltage scaling among implementations with 8 T-only, 12 T-only, and mixed cells.

(i.e., total buffer area and clock tree wirelength) compared to designs with only single-height cells. This indicates that our optimization does not incur any power and area penalties with respect to the clock tree synthesis optimization. As described in Section IV-A, we use four sets of  $(\beta, \lambda, \alpha)$  $\eta$ ) values to perform optimization and select the minimumpower outcome. In our experiments, we perform the four implementations in parallel and report the runtime corresponding to the minimum-power solution in Table IV. On the downside, we observe in our experiments that including more libraries (i.e., with mixed cell heights) typically increases the runtime of commercial SP&R tools. Furthermore, although we apply our internal timer during the placement legalization, the incremental timing analysis, placement legalization, and trial routing used for correlation also incur runtime overhead. Therefore, our mixed-height implementations are achieved at the cost of larger runtimes compared to single-height cases.

# B. Iso-Performance Power Comparison

Given that certain designs have timing violations, to achieve a fair power comparison we perform voltage scaling on each design so that all designs meet the timing constraints. We then compare power at the scaled supply voltage. In our experiments, we define scaling\_lib\_group in the PT-PX tool to enable such comparisons. Note that to compensate the slack discrepancy between Innovus and PrimeTime, we apply a constant slack shift (i.e., the difference between the WNS from *PrimeTime* versus the WNS from *Innovus*) of the entire block to correlate the post-routing worst slack values, then perform voltage scaling. When the difference between the scaled voltage and the signoff voltage is larger than 30 mV, we perform SP&R with the scaled voltage and use the smaller power value between that of the initial implementation and that of the additional implementation in our comparison.

Fig. 16 shows the iso-performance power comparison. We observe that 8 T-only designs typically have smaller power compared to 12 T-only ones. The exception of design MPEG with frequency = 1.8 GHz might be due to a larger number of buffer insertion as well as voltage scaling in the 8 T-only design to meet the performance constraints. Moreover, our optimized designs with mixed cell heights provide power reduction compared to 8 T-only and 12 T-only designs. Such power reduction mainly comes from reduced cell area and wirelength, as well as power-awareness in our partitioning optimization. Similarly to area benefit, power benefit from mixed heights over 8 T-only designs also reduces at a loose timing constraint.



Fig. 17. Layout of AES\_macro with mixed-height optimization. In gray is the placement blockage. In red are 12 T cells. In blue are 8 T cells.



Fig. 18. (a) Delay-area tradeoff and (b) delay-power tradeoff, of 8 T, 10 T, and 12 T buffers/inverters in 28 nm LP foundry libraries. Corner: (SS, 0.95 V, 125 °C). Load cap = FO4 + 20  $\mu$ m M3 wire.

#### C. Additional Validations

We further validate our optimization flow on three variants of the design AES: 1) design AES with tight timing constraints (AES\_tight); 2) design AES with blockages to model existence of macro blocks (AES\_macro); and 3) a design containing eight AES blocks (AES\_x8). For AES\_tight, we increase the maximum frequency of AES by 20% compared to that in Table IV. Results in Table V show that the mixed-height design achieves more than 10% power and area reductions, as well as better timing, compared to the 12 T-only design. Although the 8 T-only design shows smaller power and area compared to the mixed-height design, the 8 T-only design has huge timing violations. Furthermore, we implement design AES with inserted placement blockage (i.e., AES macro) to model the existence of macro blocks (as illustrated in Fig. 17). We extend our optimization flow in several ways to comprehend the existence of macro blocks: 1) we treat a grid that is occupied by a macro block as having zero cost during our DP-based partitioning step; 2) we comprehend placement blockages during our placement legalization using placement density constraints; and



Fig. 19. Pareto curves of performance-area tradeoff for implementations with 10 T-only and mixed (8 T+12 T and 8 T+10 T+12 T ) cells.



Fig. 20. Iso-performance power comparison with voltage scaling among implementations with 10 T-only and mixed (8 T + 12 T and 8 T + 10 T + 12 T) cells.

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Flow	12T	8T	mix			
Design (clk period)	AES_tight (550ps)					
#Instances	15442	16241	14691			
12T/8T	15442/0	0/16241	541/14150			
Setup WNS (ps)	-94	-230	-38			
Setup TNS (ns)	-10.981	-53.379	-2.753			
Area $(\mu m^2)$	19248	14144	16745			
WL (mm)	198	158	181			
Total power (mW)	49.5	37.1	44.6			
Runtime (min)	152	116	237			
Design (clk period)	I	AES_macro (65	50ps)			
#Instances	14679	16354	14985			
12T/8T	14679/0	0/16354	13921/1064			
Setup WNS (ps)	-26	-127	1			
Setup TNS (ns)	-0.593	-16.428	0.000			
Area $(\mu m^2)$	16610	14173	12032			
WL (mm)	167	162	169			
Total power (mW)	37.4	31.4	27.0			
Runtime (min)	124	118	206			
Design (clk period)	AES_x8 (650ps)					
#Instances	126594	114592	116095			
12T/8T	126594/0	0/114592	19865/96230			
Setup WNS (ps)	-167	-54	-52			
Setup TNS (ns)	-186.59	-32.30	-13.61			
Area $(\mu m^2)$	112075	129868	88016			
WL (mm)	1564	1629	1459			
Total power (mW)	250.6	281.8	215.0			
Runtime (min)	319	521	1515			

TABLE V RESULTS OF VARIOUS AES DESIGNS

3) we avoid overlaps with macro blocks during our cell mapping optimization. Results in Table V show more than 14% reductions in power and area with our mixed-height optimization, compared to 12 T- and 8 T-only implementations. Last, to test the scalability of our optimization flow, we generate a design containing eight AES blocks with  $\sim$ 120K instances.

#### D. Comparison to 10 T-Only Designs

We generate 10 T cell libraries by performing interpolation on timing and power tables of 12 T and 8 T libraries, which are from foundry 28LP technology.<sup>19</sup> We also generate cell LEF by scaling area of cells proportional to cell drive strengths according to area and drive strength information of 8 T and 12 T cells. In other words, we use the 8 T cell layouts from foundry technology, and scale their area accordingly to the 10 T cells' driving strength. Therefore, 10 T cells offer averaged performance-area and performance-power tradeoff points between 8 T and 12 T cells. Fig. 18 shows the delay-area and delay-power tradeoffs of 8 T, 12 T and our generated 10 T buffers and inverters. We use the methodology described in Footnote 1 to estimate cell area and delay. We estimate cell power assuming an operating frequency of 1 GHz and 10% switching activity.

Figs. 19 and 20, respectively, show performance-area Pareto curves and iso-performance comparisons between 10 T-only designs and mixed-height designs with 12 T and 8 T cells. We observe significant power and area reductions from the optimized designs with mixed heights over the 10 T-only designs. This indicates that single-height designs with an optimized performance-power/area tradeoff are not able to provide the similar performance, power, and area benefits compared to mixed cell-height designs which are able to explore a wider range of tradeoff among performance, power, and area.<sup>20</sup> Moreover, we perform mixed-height optimization with 8 T, 10 T, and 12 T cells as an example to demonstrate the scalability of our methodology to more than two cell heights. The black curves and dots in Figs. 19 and 20, respectively, show performance-area and performance-power tradeoffs of the optimized design with three cell heights. Results show similar design quality between the mixed-height solutions with

 $<sup>^{19}</sup>$ As an example, with given input slew and output load capacitance, a 10 T 1X buffer's delay is the average of the delay of an 8 T 1X buffer and the delay of a 12 T 1X buffer with the same input slew and load capacitance.

 $<sup>^{20}</sup>$ For our benchmark designs and selected clock periods, 8 T (resp. 12 T) cells typically lead to timing violations (resp. power and area overheads). We therefore consider 10 T cells to have an optimized performance-power/area tradeoff.

8 T and 12 T cells versus the solutions with 8 T, 10 T, and 12 T cells for most of the designs.<sup>21</sup> On design DES, adding 10 T cells reduces power and area. The slight power increase on design MPEG (frequency = 1.66 GHz) might come from the noise of SP&R tools as well as our optimization.

# VI. CONCLUSION

In this paper, we have proposed a novel physical design optimization flow (which includes synthesis, placement, clock tree synthesis, and routing) to mix cells with different, noninteger multiple heights in a fine-grained manner within a single placeand-route block. Our flow addresses the chicken-and-egg loop between floorplan site definition and the post-placement choice of cell heights, and correctly models (based on industry feedback from 20SOC and 16FF design experience) breaker cell overheads of the mixed-height placement. Our optimization, applied to production 12 T and 8 T libraries in a 28LP foundry technology, can achieve over 30% area and power reductions while maintaining performance, as compared to a 12 T-only design flow. Moreover, our optimized mixed-height designs can achieve significant performance increase along with area and power reductions as compared to designs with 8 T-only cells.

Since there are recent studies on mixing integral multipleheight cells (e.g., [12]), one interesting future direction might be to thoroughly compare and contrast: 1) mixing of integral multiple-height cells, which has less area penalty but coarser-grained performance, power, and area tradeoff, versus 2) mixing of noninteger cell heights, which has relatively finer-grained performance, power, and area tradeoff but area cost due to breaker cells. Furthermore, to improve the scalability of our optimization flow, additional future directions might include: 1) an improved internal timer with more accurate incremental timing estimation with respect to the golden timer and 2) a parallel optimization framework to optimize different regions of a design simultaneously.

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 $^{21}\ensuremath{\text{We}}\xspace$  understand that adding 10 T cells increases the layout solution space and would ideally provide more fine-grained tradeoffs of performance, power, and area, thus leading to potentially better solution quality. However, perhaps as a result of how we generate 10 T cells (i.e., by averaging the performance, power, and area of 8 T and 12 T cells), adding 10 T cells does not significantly expand the available area-delay or power-delay tradeoff (which can also be observed from Fig. 18), nor does it lead to improved design quality in our experiments. Another explanation could be the not-unexpected "limit of incremental benefit" that one would expect from additional available cell heights-similar to how added VT flavors or added cell sizes ("rich libraries") also eventually show diminishing or zero incremental returns.

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