

Multilevel Circuit Partitioning¹

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Abstract

Recent work [2] [5] [11] [12] [14] has illustrated the promise of *multilevel* approaches for partitioning large circuits. Multilevel partitioning recursively clusters the instance until its size is smaller than a given threshold, then unclusters the instance while applying a partitioning refinement algorithm. Our multilevel partitioner uses a new technique to control the number of levels in the matching-based clustering phase and also exploits recent innovations in classic iterative partitioning [7] [10]. Our heuristic outperforms numerous existing bipartitioning heuristics, with improvements ranging from 6.9 to 27.9% for 100 runs and 3.0 to 20.6% for just 10 runs (while also using less CPU time).

1 Introduction

A netlist hypergraph $H(V, E)$ has n modules $V = \{v_1, v_2, \dots, v_n\}$; a net $e \in E$ is defined to be a subset of V with size greater than one. A *bipartitioning* $P = \{X, Y\}$ is a pair of disjoint clusters (i.e., subsets of V) X and Y such that $X \cup Y = V$. The *cut* of a bipartitioning $P = \{X, Y\}$ is the number of nets which contain modules in both X and Y , i.e., $cut(P) = |\{e \mid e \cap X \neq \emptyset, e \cap Y \neq \emptyset\}|$. Let $A(v)$ denote the area of $v \in V$ and let $A(S) = \sum_{v \in S} A(v)$ denote the area of a subset $S \subseteq V$. Given a balance tolerance r , the *min-cut bipartitioning problem* seeks a solution $P = \{X, Y\}$ that minimizes $cut(P)$ subject to $\frac{A(V)(1-r)}{2} \leq A(X), A(Y) \leq \frac{A(V)(1+r)}{2}$.

The standard bipartitioning approach is iterative improvement based on the Kernighan-Lin algorithm, which was later improved by Fiduccia-Mattheyses (FM) [8]. The FM algorithm begins with some initial solution $\{X, Y\}$ and proceeds in a series of *passes*. During a pass, modules are successively moved between X and Y until each module has been moved exactly once. Given a current solution $\{X', Y'\}$, the previously unMOVED module $v \in X'$ (or Y') with highest *gain* ($= cut(\{X' - v, Y' + v\}) - cut(\{X', Y'\})$) moved from X' to Y' . After each pass, the best solution $\{X', Y'\}$ observed during the pass becomes the initial solution for a new pass, and the passes terminate when a pass does not improve the initial solution. FM has been widely adopted due to its short runtimes and ease of implementation.

Iterative approaches dominate both the VLSI CAD literature and industry practice for several reasons. They are generally intu-

itive (the obvious way to improve a given solution is to repeatedly make it better via small changes), easy to describe and implement, and relatively fast. Hence, much work has sought to improve upon the classic FM algorithm [7] [10] [15]. Other works attempt to use iterative improvement as an engine inside other algorithmic approaches such as large-scale Markov chains [9], two-phase clustering [4] or multilevel clustering [5] [12] [11] [14] (see [3] for a survey on partitioning and clustering techniques).

This paper proposes a new multilevel circuit partitioning algorithm that is motivated by the success of multilevel partitioners [12] [14] in the scientific computing community. We have added two key ingredients to the functionality of our partitioner, which significantly improve performance: (i) we use the CLIP algorithm of [7] within our FM implementation, and (ii) we use a matching based clustering that halts prematurely so that more than $\frac{n}{2}$ clusters are generated. This causes the multilevel coarsening to proceed more slowly, a major source of our superior solution quality.

2 The Multilevel Partitioning Paradigm

As problem sizes grow larger, the performance of iterative improvement approaches such as FM tend to degrade. The technique of *clustering* or *coarsening* is commonly used to deal with increasing problem sizes. The netlist modules are divided into many small clusters, and these clusters form the new nodes of a smaller, coarser netlist. Iterative improvement can then be run on this coarsened netlist. Since multilevel partitioning is based on this concept, we now present some definitions to make these ideas more rigorous.

Definition 1: A clustering $P^k = \{C_1, C_2, \dots, C_k\}$ of H_i induces the *coarser netlist* $H_{i+1}(V_{i+1}, E_{i+1})$ where $V_{i+1} = \{C_1, C_2, \dots, C_k\}$. For every $e \in E_i$, the net e^* belongs to E_{i+1} where $e^* = \{C_h \mid e \cap C_h \neq \emptyset\}$, unless $|e^*| = 1$. In other words, e^* spans the set of clusters containing modules of e .

Definition 2: Suppose that H_{i+1} was induced from H_i by the clustering $P^k = \{C_1, C_2, \dots, C_k\}$. The *projection* of the bipartitioning solution $P_{i+1} = \{X_{i+1}, Y_{i+1}\}$ of H_{i+1} onto H_i is the solution $P_i = \{X_i, Y_i\}$ where $X_i = \{v \in V_i \mid \exists C_h \in P^k, v \in C_h, C_h \in X_{i+1}\}$ and $Y_i = \{v \in V_i \mid \exists C_h \in P^k, v \in C_h, C_h \in Y_{i+1}\}$. The process of projecting P_{i+1} to P_i is called *uncoarsening*.

Clustering has often been applied within a “two-phase” methodology. First a clustering P^k of H_0 is generated, then this clustering is used to induce the coarser netlist H_1 from H_0 . FM is then run once on H_1 to yield the bipartitioning P_1 and this solution P_1 is projected to a new bipartitioning P_0 of H_0 . Finally, FM is run a second time on H_0 using P_0 as its initial solution. This second FM run is called a *refinement* step, which refers to the improvement of an initial good solution via local moves and swaps.

The “two-phase” approach can be extended to a *multilevel* approach by using as many phases as are desired. In a multilevel algorithm, a clustering of H_0 is used to induce the coarser netlist H_1 , then a clustering of H_1 induces H_2 , etc. until the most coarsened

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netlist H_m is constructed. A bipartitioning solution $P_m = \{X_m, Y_m\}$ is found for H_m (e.g., via FM) and this solution is then projected to $P_{m-1} = \{X_{m-1}, Y_{m-1}\}$. P_{m-1} is then refined, e.g., by FM post-processing. The uncoarsening process continues until a refined partitioning of H_0 is obtained.

Multilevel partitioning offers several advantages over two-phase FM. First, the single coarsening step of two-phase FM can make H_1 too coarse a representation of H_0 . Multilevel partitioning enables coarsening proceeds more slowly, giving the iterative engine more opportunities for refinement. Second, multilevel partitioning can be extremely efficient if a fast clustering and refinement strategy is used. Refinement for each netlist H_i typically requires only a few FM passes to converge since it begins with a high-quality initial solution. Finally, refinement proceeds with progressively larger netlists, implying that the number of modules moved during a single step of FM becomes progressively smaller. This permits the refinement algorithm to avoid bad local minima via big steps at high levels, while still being able to find a good final solution via detailed refinement at low levels.

Work in multilevel partitioning [12] [14] [18] has been especially prominent in the scientific computing literature for partitioning finite-element graphs. Hendrickson and Leland [12] developed a very efficient multilevel partitioning algorithm, included in their Chaco package. They use random matching [4] to perform clustering and multi-way FM to do refinement, with several modifications to reduce runtime: (i) the algorithm can terminate before a pass is completed if further improvement appears unlikely; (ii) gains are saved after a pass is completed so that only moved modules and their neighbors need to have their gains recomputed before the next pass; and (iii) an efficient boundary refinement scheme is used wherein only vertices incident to cut edges are inserted into the data structure, with gains for other vertices computed only on an “as needed” basis. Metis, another multilevel partitioning package targeted to finite-element graphs, was developed by Karypis and Kumar [14]. As in [12], boundary schemes and early pass termination are used, along with many different algorithms for clustering, initial partitioning and refinement which allow experiments with various combinations of options. One of the Metis coarsening schemes uses a greedy weighted matching algorithm, upon which our coarsening scheme is based.

In the VLSI CAD community, previous multilevel works include [2] [5] [11]. The authors of [2] adapt Metis to partition netlist hypergraphs and use a genetic algorithm to obtain more stable solution quality. The authors of [5] apply clique-finding clustering as the coarsening step for multilevel bipartitioning. Finally, the authors of [11] give a detailed study of multilevel FPGA partitioning, exploring various schemes for technology mapping, clustering, partitioning the coarsest graph, and uncoarsening in one or multiple steps. Their final algorithm uses simple connectivity-based clustering and iterative improvement with two or three levels of lookahead.

3 A New Multilevel Algorithm

Figure 1 describes ML, our multilevel algorithm for partitioning netlist hypergraphs. The algorithm accepts a netlist H_0 as input along with two user parameters T and R . T sets a threshold that bounds the number of modules in the smallest netlist H_m , and R is a parameter used by the *Match* coarsening algorithm explained below. The variable m denotes the number of levels used during coarsening, and the variables P^k and P_i denote intermediate clustering and bipartitioning solutions respectively.

The first four steps in Figure 1 form the coarsening phase. As long as the number of modules in H_i is greater than T , *Match* is used to form a clustering P^k of H_i . Following Definition 1, procedure *Induce* takes a netlist H_i and a clustering P^k and constructs the

ML Multilevel Algorithm	
Input:	$H_0(V_0, E_0) \equiv$ Netlist hypergraph $T \equiv$ Coarsening threshold $R \equiv$ Matching ratio
Variables:	$m \equiv$ Number of levels $P^k \equiv$ Intermediate clusterings $P_i, 1 < i \leq m \equiv$ Intermediate bipartitionings
Output:	$P_0 = \{X_0, Y_0\} \equiv$ Final bipartitioning
<ol style="list-style-type: none"> 1. while $V_i > T$ do 2. $P^k = \text{Match}(H_i, R)$. 3. $H_{i+1}(V_{i+1}, E_{i+1}) = \text{Induce}(H_i, P^k)$. 4. Set $i = i + 1$. 5. Let $m = i$. $P_m = \text{FMPartition}(H_m, \text{NULL})$. 6. for $i = m - 1$ down to 0 do 7. $P_i = \text{Project}(H_{i+1}, P_{i+1})$. 8. $P_i = \text{FMPartition}(H_i, P_i)$. 9. return P_0. 	

Figure 1: The ML Multilevel Algorithm.

new netlist H_{i+1} induced by P^k (while preserving module areas). Step 5 constructs a bipartitioning of H_m using the *FMPartition* procedure, and Steps 6-8 form the uncoarsening phase. Following Definition 2, the *Project* procedure takes a netlist H_{i+1} and a bipartitioning P_{i+1} of H_{i+1} and constructs the projection of P_{i+1} onto P_i of H_i . The projected solution is then refined via *FMPartition*, and uncoarsening proceeds until a refined partitioning P_0 of H_0 is obtained; this solution is returned in Step 9. We now discuss the procedures *Match* and *FMPartition* in more detail.

We coarsen via a linear time “heavy-edge” matching similar in spirit to [14]. The *Match* algorithm starts by randomly permuting the modules and then visits each in turn. For a given module $v = v_{\pi(j)}$, *Match* tries to find the unmatched module w (i.e., a module that has not yet been assigned to a cluster) with highest connectivity to v , where the connectivity between v and w is defined as

$$\text{conn}(v, w) = \frac{1}{A(v) \cdot A(w)} \sum_{e \in \{e | v \in e, w \in e\}} \frac{1}{|e|}$$

The term $\frac{1}{|e|}$ emphasizes nets with fewer modules, and the term $\frac{1}{A(v) \cdot A(w)}$ gives preference to matching modules with smaller areas to help prevent cluster sizes from becoming too unbalanced. If such a w can be found then v and w are matched together, i.e., they form a new cluster. If no unmatched w exists (i.e., all of the neighbors of v are matched), then v is assigned to its own cluster. When computing the *conn* function, we ignore nets with more than ten modules to reduce runtimes.

The matching algorithms of [4] [12] [14] seek maximal matchings, which generally forces the ratio of $|V_{i+1}|$ to $|V_i|$ to be $\frac{1}{2}$. We believe that maximal matching can result in too few levels; a slower coarsening gives the refinement algorithm more opportunities to construct better solutions. Further, slower coarsening reduces the differences between successively coarser netlists H_i and H_{i+1} which implies that iterative refinement of H_i will take fewer passes to converge. To control the speed of coarsening, *Match* takes a parameter $0 \leq R \leq 1$, called the *matching ratio*, specifying the fraction of modules to be matched. For example, when $R = 1$ a maximal matching is sought, but when $R = 0.5$ the matching continues only until half of the modules are matched (each remaining unmatched module is assigned to its own cluster).

Figure 2 shows the *Match* coarsening procedure. The while loop in Step 2 continues as long as the ratio of matched modules to the total number of modules is less than R or until all the modules have been examined. Step 3 adds the current module $v_{\pi(j)}$ to

Procedure Match	
Input:	$H_i(V_i, E_i) \equiv$ Netlist hypergraph $R \equiv$ Matching ratio
Variables:	$k \equiv$ Number of clusters $\pi \equiv$ Permutation of V_i $nMatch \equiv$ Number of matched modules $j \equiv$ Current module index $w \equiv$ Matched module
Output:	$P^k \equiv$ Clustering of H_i
1. Construct random permutation π of V_i . Set $nMatch = k = 0, j = 1$. 2. while $\frac{nMatch}{ V_i } < R$ and $j < V_i $ do 3. Set $k = k + 1$. Add $v_{\pi(j)}$ to cluster C_k . 4. Find unmatched $w \in V_i$ that maximizes $comm(v_{\pi(j)}, w)$. 5. if such a w exists then add w to C_k and set $nMatch = nMatch + 2$. 6. Set $j = j + 1$. 7. while $j < V_i $ do 8. if $v_{\pi(j)}$ is unmatched then Set $k = k + 1$. Assign $v_{\pi(j)}$ to cluster C_k . 9. Set $j = j + 1$. 10. return $P^k = \{C_1, C_2, \dots, C_k\}$.	

Figure 2: The Match Procedure.

the current cluster, and Step 5 also adds the module w to the cluster if a matched module w can be found in Step 4. When Step 7 is reached, matching is complete. Each remaining unmatched module is assigned to its own cluster in Steps 7-9, and the final clustering obtained is returned in Step 10. Assuming constant degree bounds on the modules and size bounds on the nets, *Match* can be implemented in linear time through careful exploration of the neighbors of $v_{\pi(j)}$ in Step 4.

Our refinement algorithm *FMPartition* takes a netlist H_i and an initial partitioning solution P_i as input, and returns a refined partitioning of P_i . Since large nets can slow down FM, nets with more than 200 modules are ignored (these nets are reinserted when measuring solution quality). If the initial partitioning passed in is *NULL*, as in Step 5 of Figure 1, then a random starting solution is constructed. Our partitioner uses FM with a LIFO bucket scheme [10] and may also use CLIP [7] if desired. CLIP uses the idea of infinite weight tie-breaking, e.g., suppose that moving module v_i increases the gain of v_j by one. Instead of increasing the gain by just one, it could be increased by two, five, ten, etc. The authors of [7] actually propose to increase the gain by an infinite factor and accomplish this by initializing all cells to the zero gain bucket in order of their true gains. Experiments in [7] show that CLIP averages 18% improvement over FM (both using a LIFO bucket scheme).

4 Experimental Results

We ran our partitioner on 23 of the standard benchmarks from the CAD Benchmarking Laboratory (ftp.cbl.ncsu.edu or visit our web site at <http://vlsicad.cs.ucla.edu/>). The characteristics for these test cases can be found in e.g., [2] [7]. We report bipartitioning results for unit module areas with $r = 0.1$. The FM- and CLIP-based implementations for our ML algorithm are denoted by ML_F and ML_C respectively. For all experiments, the coarsening threshold was set to $T = 35$ modules.

Our first experiments study the effects of varying the matching ratio parameter R : we ran ML 100 times for each test case with R values 1.0, 0.5 and 0.33. Due to space limitations, Table 1 includes only the data for the 12 largest test cases (see [1], or our website <http://vlsicad.cs.ucla.edu/>, for the entire data set).

For all the benchmarks, the difference among minimum cuts for various values of R is less than 2%, except for the largest benchmarks. However, the minimum cuts significantly improve for the very largest benchmarks (particularly *golem3*) for smaller values of R . Over all 23 benchmarks, ML_F yielded 7.9% and 9.5% respective reduction of *average* cut size for $R = 0.5$ and $R = 0.33$ over $R = 1.0$, while ML_C yielded 6.9% and 7.6% lower average cuts. For $R = 1.0$, the average cuts of ML_C were 5.5% lower than those of ML_F . We choose to use ML_C with $R = 0.5$ for comparing with other algorithms since the gains of CLIP over FM and $R = 0.5$ over $R = 1.0$ are significant, while reducing R to 0.33 does not seem worth the extra runtime [1].

Alg	Test Case	MIN			AVE		
		1.0	0.5	0.33	1.0	0.5	0.33
ML_C	s9234	41	40	40	48	45	45
	biomed	83	83	83	92	91	91
	s13207	60	55	58	76	71	68
	s15850	43	44	43	59	56	57
	ind2	174	164	167	197	196	292
	ind3	248	243	244	274	276	276
	s35932	40	41	42	46	45	46
	s38584	48	47	47	58	52	52
	avqsm1	139	133	132	194	159	156
	s38417	53	50	50	82	72	68
	avqlrg	144	130	131	200	163	157
	golem3	1663	1348	1347	2026	1462	1421
ML_F	s9234	40	40	40	50	47	47
	biomed	86	83	83	103	96	94
	s13207	58	55	58	77	72	71
	s15850	43	43	42	63	58	59
	ind2	168	171	169	213	207	207
	ind3	243	243	241	275	277	275
	s35932	41	42	42	46	48	49
	s38584	49	48	47	77	56	57
	avqsm1	133	128	128	182	147	148
	s38417	50	49	49	66	56	56
	avqlrg	140	128	129	183	148	148
	golem3	1661	1346	1340	2006	1465	1413

Table 1: Minimum cut, average cut and total CPU time obtained for 100 runs of ML_C for different values of the matching ratio R .

Many works which present bipartitioning results for unit module areas and size constraints corresponding to $r = 0.1$. Table 2 compares the cuts and obtained by ML_C with $R = 0.5$ for 100 and 10 runs to seven such algorithms in the literature. GM [2] and HB [11] are multilevel approaches, PARABOLI (PB) [19] uses linear placement techniques, GFM_l is a two-phase gradient FM approach [16], and CL-LA3_f (CLIP with lookahead level 3), CD-LA3_f (CDIP with lookahead level 3) and CL-PR_f (CLIP with PROP gain calculation) [7] are three modifications to the FM engine. More complete comparisons with other algorithms whose results are subsumed by these works can be found in [1] or our website.

The last two rows of the table respectively give the percent improvements of ML_C with 100 runs, and ML_C with 10 runs, over the other algorithms in terms of cut size. ML_C with 100 runs averages between 6.9% and 27.9% improvement in cut sizes. Even when limiting ML_C to just 10 runs, we still obtain between 3.0% and 20.6% improvement over the other algorithms.¹

From Table 1 we see that the *average* cut obtained for *golem3* was 1465, which is still significantly better than the best known result. The table also reports the total time required for 10 runs of

¹For 10 and 100 runs of ML_C , we respectively averaged 19.1% and 21.9% improvement over our implementation of LSMC [9] (22 test cases), 6.5% and 11.1% improvement over GFM [16] (13 test cases), and -1.7% and 2.4% improvement over PANZA [17] (9 test cases). Note that PANZA does not report results some of the largest benchmarks (e.g., *industry2*, *avqsmall*, *avqlarge*, and *golem3*) for which our approach has been particularly successful.

Test Case	Cut size									CPU Time (s)						
	ML _C (100)	ML _C (10)	GM	HB	PB	GFM _i	CL-LA3 _f	CD-LA3 _f	CL-PR _f	ML _C (10)	GM	PB	GFM _i	CL-LA3 _f	CD-LA3 _f	CL-PR _f
balu	27	27	27		41	28	27	27	27	17	14	16	25	32	31	34
bm1	47	51	48				51	47	47	18	12			37	47	36
prim1	47	52	47		53	51	51	47	51	18	12	18	25	36	48	37
test04	48	49	49				49	48	52	41	21			81	106	114
test03	56	58	62				56	57	57	47	23			88	107	95
test02	89	92	95				91	89	87	45	26			99	124	109
test06	60	60	94				60	60	60	55	32			50	55	175
struct	33	33	33		40	36	33	36	33	35	27	35	32	45	54	75
t05	71	72	104				80	74	77	74	46			141	162	188
19ks	106	108	106				104	104	104	84	39			178	216	219
prim2	139	145	142		146	139	142	151	152	90	53	137	61	167	210	353
s9234	40	41	43	45	74	44	45	44	42	97	58	490	186	175	270	264
biomed	83	84	83		135	92	83	83	84	172	95	711	371	231	362	572
s13207	55	55	70	62	91	61	66	69	71	155	102	2060	397	220	429	380
s15850	44	56	53	46	91	46	71	59	56	189	114	1731	530	267	543	576
ind2	164	174	177		193	175	200	182	192	502	245	1367	819	1129	1453	2127
ind3	243	243	243		267	244	260	243	243	667	299	761	861	1419	1944	1920
s35932	41	42	57	46	62	44	73	73	42	427	266	2627	1088	463	964	1085
s38584	47	48	53	52	55	54	50	47	51	490	397	6518	3463	748	1339	1950
avqsm	128	134	144		224		129	139	144	603	328	4099		1260	2507	2082
s38417	49	50	69		49	62	70	74	65	496	281	2042	1062	811	1733	1690
avqlrg	128	131	144		139		127	137	143	666	417	4135		1430	3145	2126
golem3	1346	1374	2111		1629					10483	450	10823				
%imprv	x		16.9	9.5	27.9	7.8	9.2	11.5	6.9							
%imprv		x	8.4	3.0	20.6	3.6	6.0	7.9	5.2							

Table 2: Cut size and CPU comparisons of ML_C (for 100 runs and for 10 runs) with seven other bipartitioning algorithms.

ML_C on a Sun Sparc 5 (85Mhz). The runtimes for GM, CL-LA3_f, CD-LA3_f and CL-PR_f are also given for this machine. PB and GFM(GFM_i) runtimes are reported for a Dec 3000 Model 500 AXP and a Sun Sparc 10, respectively. Although runtimes across different platforms are not directly comparable, the 10 runs of ML_C use less runtime than any of the other algorithms except GM. It seems that if a reasonably high quality result is desired in a few seconds, then GM is appropriate; however, if a bit more CPU time can be afforded, ML_C is the better choice. Overall, our multilevel algorithm with a CLIP engine and $R = 0.5$ provides excellent cut results compared to previous algorithms while requiring only moderate CPU resources.

As the golem3 data shows, multilevel partitioning is best suited for very large instances; however, the lack of public test cases with more than 25,000 modules makes this difficult to illustrate. Indeed, the algorithms have begun to converge to the same cut size for most of the smaller benchmarks; without newer, larger test cases, it will be difficult to recognize improvement or innovation from any new partitioner.

Our current efforts seek to speed up our approach (e.g., via boundary refinement schemes and propagation of gain data down the hierarchy [12] [14]) while maintaining high solution quality. We have also integrated a 4-way partitioning version of ML_C to yield an excellent quadrisection-based placement tool [13].

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