NN-Steiner: A Neural-Algorithmic Approach for the Rectilinear Steiner Minimum Tree Problem

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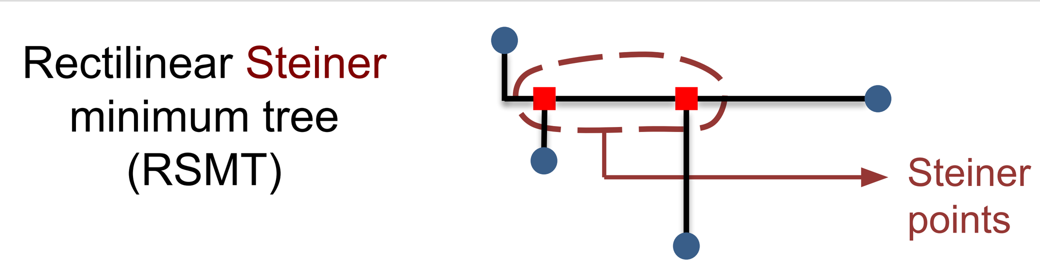
Overview

Main Idea: Combine algorithmic insights with neural networks

- **Motivation:** The rectilinear Steiner minimum tree (RSMT) problem, which is NP-hard, is fundamental to IC layout design
- Arora's algorithm for RSMTs achieves state-of-the-art (SOTA) theoretical guarantees, too costly for practice
- Our approach: NN-Steiner
  - Implementation of Arora's celebrated polynomial-time approximation scheme (PTAS) algorithm via a mixed-algorithmic-NN approach
  - **Replaces costly sub-algorithmic components with learning**, while keeping the DP framework
- NN-Steiner advantages:
  - Practical while still leveraging algorithmic insights
  - Uses bounded-size neural networks, thus efficient and effective to train
  - Learned sub-algorithmic components generalize to larger point sets than seen in training

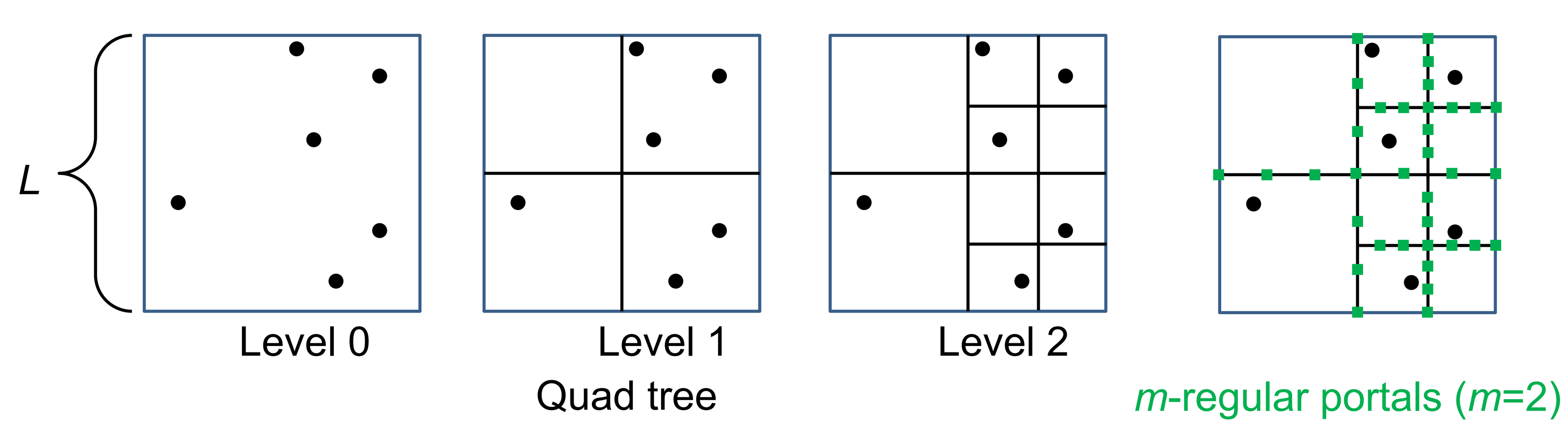
Rectilinear Steiner Minimum Trees

Problem Statement:
Input: Point set $V \subset \mathbb{R}^2$
Output: Tree T with vertices $U = V \cup S$ and minimum length under ℓ_1 distance



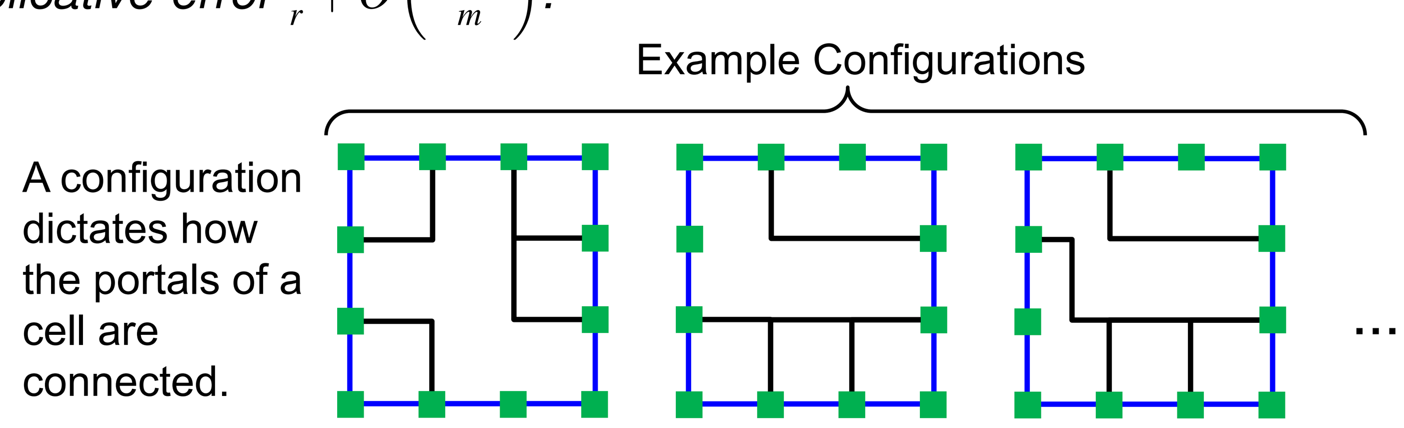
Arora's Algorithm

Key components:



Definition. *A tree is (m,r)-light if it crosses each side of each quad-tree cell at most r times, always at an m-regular portal.*

Theorem. *(m,r)-light trees approximate the length of RSMTs to within multiplicative-error $\frac{4}{r} + O\left(\frac{4logL}{m}\right)$.*



Algorithm [1]:

1. Construct quad tree
2. Base step: compute cost for each configuration of each quad-tree leaf
3. Dynamic programming step: compute the configuration costs for each quad-tree cell using the costs of its child cells
4. Combine costs at the quad-tree root to find the minimum-cost (m,r)-light tree

Problem: Number of configurations is bounded, but too large in practice
Solution:

- Keep the DP framework
- Replace brute-force computation of poartal configuration costs with neural networks

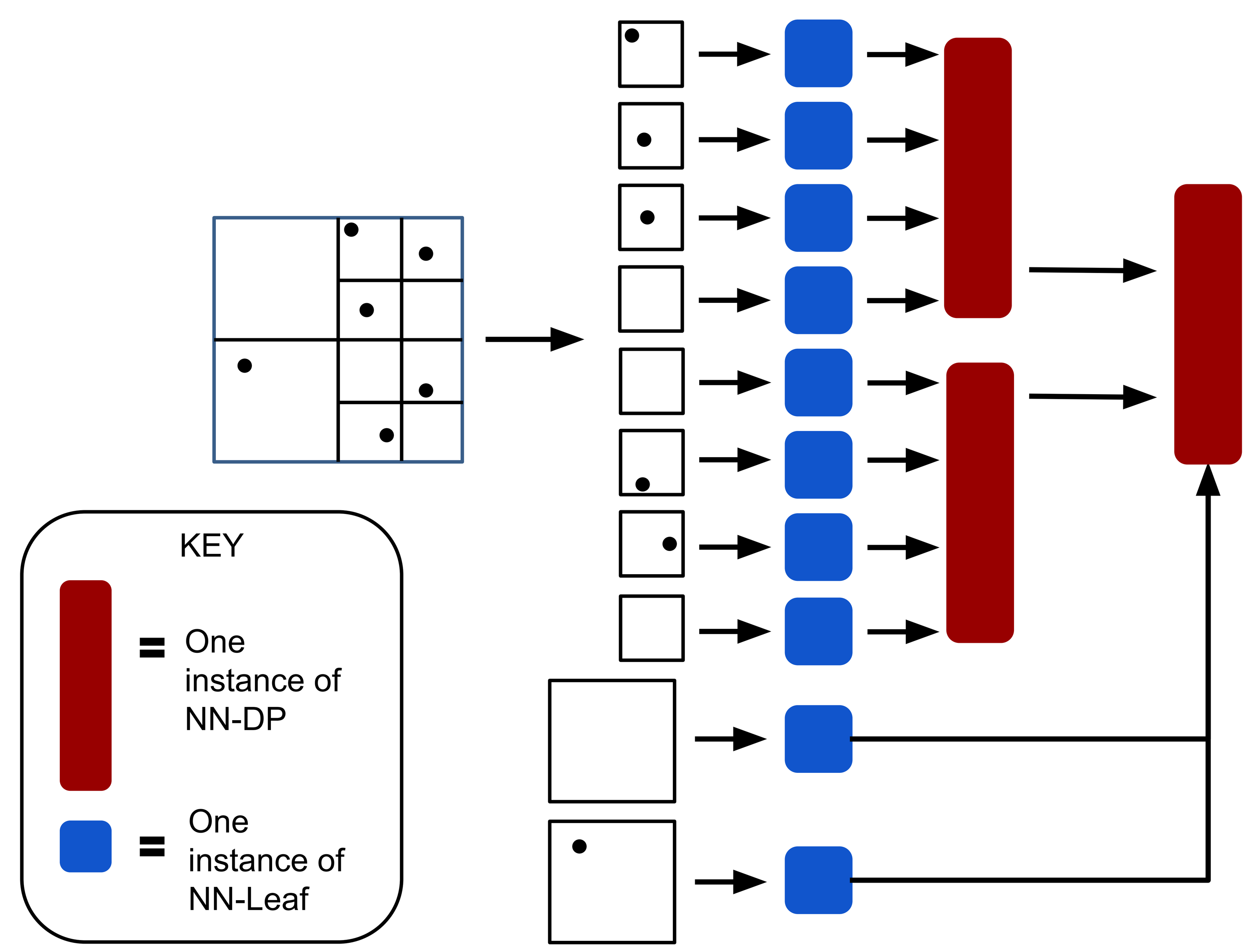
NN-Steiner

NN-Steiner is a mixed neural-algorithmic approach based on Arora's SOTA algorithm

Four bounded-size NN components, each called multiple times, are used:

- **NN-Leaf:** a cell's terminal and portal locations $\rightarrow$ an encoding of the cell's configuration costs
- **NN-DP:** output of 4 instances of NN-Leaf or NN-DP $\rightarrow$ an encoding of the cell's configuration costs
- **NN-Top:** output of top-level NN-DP $\rightarrow$ portal likelihoods
- **NN-Retrieve:** output of NN-DP and edge portal likelihoods $\rightarrow$ portal likelihoods

Thresholding the portal likelihoods at $t = .95$ yields the set of Steiner points S.



- NN components do not depend on problem size
- NN-Steiner generalizes to different problem sizes
- We can restrict training to fixed-sized problems!

Experimental Results

Algorithm \ Num. Points	50	100	200	500	800	1000	2000	5000
NN-Steiner	2.10	1.38	0.74	**-0.67**	**-1.11**	**-1.43**	**-2.44**	**-2.99**
REST [4]	**-0.17**	1.07	7.40	22.67	35.16	42.52		
FLUTE [2]	0.00	**0.00**	**0.00**	0.00	0.00	0.00	0.00	0.00
Geosteiner [3] (exact)	-0.55	-1.23	-2.25	-3.71	-4.43	-4.78		

Results are an average of 100 point sets sampled from a uniform distribution, and are reported as a percentage length-difference compared to FLUTE. REST is the SOTA NN algorithm.

Results show **NN-Steiner generalizes to large point sets**, despite training on point sets of size 180.

Acknowledgments

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References

[1] S. Arora. Polynomial time approximation schemes for Euclidean traveling salesman and other geometric problems. *Journal of the ACM (JACM)*, 45(5):753–782, 1998.

[2] C. Chu and Y.-C. Wong. FLUTE: Fast lookup table based rectilinear Steiner minimal tree algorithm for VLSI design. *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, 27(1):70–83, 2007.

[3] D. Juhl, D. M. Warme, P. Winter, and M. Zachariasen. The GeoSteiner software package for computing Steiner trees in the plane: an updated computational study. *Mathematical Programming Computation*, 10(4):487–532, 2018.

[4] J. Liu, G. Chen, and E. F. Young. REST: Constructing rectilinear Steiner minimum tree via reinforcement learning. In *Proccedings of the 58th ACM/IEEE Design Automation Conference (DAC)*, pages 1135–1140, 2021.