# Lifetime Estimation

## Problem Statement
An analytical model to estimate lifetime of interconnects under overdrive and nominal modes of operations.

### Input: (1) Nominal MTTF ($MTTF_{nom} = 10$years), $V_{dd}$ ($V_{dd,nom}$), max. temperature ($T_{max,nom}$), RMS current density limit ($J_{rms,nom}$), duty cycle ($d_{nom}$), (2) Overdrive $V_{dd}$ ($V_{dd,ov}$), duty cycle ($d_{ov}$), and (3) Technology parameters width ($w$), height ($h$) of interconnect, operating modes = {normal, overdrive}.

### Subject to: Area of void due to electromigration does not exceed cross-section of interconnect.

## Terminologies
Table 1 describes the terms used in the derivation

<table>
<thead>
<tr>
<th>Term</th>
<th>Typical Value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w$</td>
<td>From tech. LEF</td>
<td>width of interconnect</td>
</tr>
<tr>
<td>$h$</td>
<td>From tech. LEF</td>
<td>height of interconnect</td>
</tr>
<tr>
<td>$MTTF_{nom}$</td>
<td>10years</td>
<td>nominal MTTF from foundry</td>
</tr>
<tr>
<td>$J_{rms,nom}$</td>
<td>From tech. LEF</td>
<td>RMS current density limit at $MTTF_{nom}$</td>
</tr>
<tr>
<td>$J_{peak,nom}$</td>
<td>From tech. LEF</td>
<td>Peak current density limit at $MTTF_{nom}$</td>
</tr>
<tr>
<td>$T_{nom}$</td>
<td>105°C</td>
<td>signoff temperature at $MTTF_{nom}$</td>
</tr>
<tr>
<td>$A_{c}$</td>
<td>-</td>
<td>cross-section area of interconnect</td>
</tr>
<tr>
<td>$q$</td>
<td>-</td>
<td>proportionality constant</td>
</tr>
<tr>
<td>$A$</td>
<td>geometry and material constant of interconnect</td>
<td></td>
</tr>
<tr>
<td>$E_a$</td>
<td>0.7eV</td>
<td>activation energy of interconnect metal ions</td>
</tr>
<tr>
<td>$k$</td>
<td>-</td>
<td>Boltzmann’s constant</td>
</tr>
<tr>
<td>$A_{op}$</td>
<td>[normal, overdrive]</td>
<td>mode of operation</td>
</tr>
<tr>
<td>$J_{rms,op}$</td>
<td>-</td>
<td>RMS current density limit at op mode of operation</td>
</tr>
<tr>
<td>$MTTF_{op}$</td>
<td>-</td>
<td>MTTF at op mode of operation</td>
</tr>
<tr>
<td>$POH_{op}$</td>
<td>-</td>
<td>power on hours at op mode of operation</td>
</tr>
<tr>
<td>$d_{op}$</td>
<td>-</td>
<td>duty cycle for op mode of operation</td>
</tr>
<tr>
<td>$T_{op}$</td>
<td>-</td>
<td>peak temperature (K) for op mode</td>
</tr>
<tr>
<td>$A_{op}$</td>
<td>-</td>
<td>switching activity for op mode</td>
</tr>
<tr>
<td>$POH_{nom}$</td>
<td>-</td>
<td>power-on-hours for normal mode of operation</td>
</tr>
<tr>
<td>$POH_{nom}$</td>
<td>-</td>
<td>power-on-hours for normal mode of operation</td>
</tr>
<tr>
<td>$T_{nom}$</td>
<td>-</td>
<td>peak temperature (K) for nominal operation</td>
</tr>
<tr>
<td>$A_{nom}$</td>
<td>-</td>
<td>switching activity for normal operation</td>
</tr>
<tr>
<td>$t_{LIFE}$</td>
<td>-</td>
<td>lifetime of interconnect</td>
</tr>
<tr>
<td>$t_{nuc}$</td>
<td>-</td>
<td>NG model time for void nucleation</td>
</tr>
<tr>
<td>$t_{g}$</td>
<td>-</td>
<td>NG model time for void growth</td>
</tr>
<tr>
<td>$M$</td>
<td>-</td>
<td>maximum number of overdrive mode cycles</td>
</tr>
<tr>
<td>$N$</td>
<td>-</td>
<td>maximum number of nominal mode cycles</td>
</tr>
</tbody>
</table>

### Table 1: ORION_NEW model for Instances

**Derivation:**

Let the critical cross-section area beyond which interconnect breaks be given as [11]:

$$A_{c} = w \cdot h$$  \hspace{1cm} (1)

The nominal MTTF is related $A_{c}$ as [3]:

$$MTTF_{nom} = q \cdot A_{c}$$  \hspace{1cm} (2)

where $q$ is a proportionality constant for $MTTF_{nom} = 10$years.

From Black’s Equation [4] on $MTTF_{nom}$, we have:

$$MTTF_{nom} = \frac{A}{J_{rms,nom}^2} \cdot \exp \left( \frac{E_a}{k} \cdot \frac{1}{T_{max,nom}} \right)$$

where $A$ is a geometry and material constant, $E_a$ is the activation energy of interconnect material and $k$ is Boltzmann’s constant.

Although older studies [7, 13] show that Black’s equation (Eqn 3) is good for both DC as well as pulsed-DC stress conditions, recent studies [3, 5, 9, 11] suggest that Black’s equation need to be modified to account for varied degradation of MTTF under strong and weak current densities. Lloyd et al. [5] suggests the nucleation and growth (NG) model of void is a better fit of the electromigration model to silicon measurement data [3, 11]. This NG model is also suitable for our experiments because we operate our circuit in overdrive and normal modes for varying durations. the signoff criterion for each of these modes has different current density limits. Using this model we can simulate the history effect of failure due to EM as a function of temperature, current density limit and duty cycle of the overdrive or stress condition.

The time to failure ($t_{LIFE}$) of an interconnect as given by the NG model [5] is divided into two phases:

$$t_{LIFE} = t_{nuc} + t_{g}$$  \hspace{1cm} (4)

where $t_{nuc}$ is the void nucleation time, that is, when the interconnect begins with having no void till the point where a void will begin to form and $t_{g}$ is the void evolution time till its cross-section area becomes $A_{c}$ as given by Eqn (1). Void nucleation takes place when the current density is low and evolution takes place at high current densities [3, 5]. Hence, $t_{nuc}$ corresponds to the duration of nominal mode of operation whereas $t_{g}$ corresponds to the duration of overdrive mode of operation.

The interconnect is always designed to meet overall lifetime $t_{LIFE}$ of $MTTF_{nom}$ under nominal operation. For the overdrive mode, an equivalence needs to be established between $t_{LIFE}$ and the lifetime under overdrive operation. Acceleration factor ($AF$) [2, 12] is used for this equivalence and is given by:

$$AF(T_{max,nom}, T_{op}) = \frac{MTTF_{nom}}{MTTF_{op}} = \frac{\alpha_{op}}{\alpha_{nom}} \cdot \exp \left( \frac{E_a}{k} \cdot \left[ \frac{1}{T_{max,nom}} - \frac{1}{T_{op}} \right] \right)$$  \hspace{1cm} (5)
where \( T_{op} \) is the peak temperature at \( op \) mode of operation, \( MTTF_{op} \) is the MTTF at \( op \) mode of operation (\( MTTF_{op} = MTTF_{nom} \cdot d_{c_{op}} \)), \( d_{c_{op}} \) is the duty cycle for \( op \) mode, \( \alpha_{op} \) is the switching activity at \( op \) mode and \( \alpha_{nom} \) is the switching activity for the nominal mode.

We define power-on-hours (\( POH_{op} \)) as the effective number of hours consumed out of \( t_{LIFE} \) under a given operating condition. For example, \( POH_{nom} \) is the effective number of hours consumed out of \( t_{LIFE} \) under nominal operation whereas \( POH_{ov} \) is the effective number of hours consumed out of \( t_{LIFE} \) under overdrive operation. So, from Equation (4),

\[
t_{muc} = POH_{nom} \quad \text{and} \quad t_{g} = POH_{ov}.
\]

So, we can calculate \( AF \) for each mode of operation as:

\[
AF_{nom}(T_{max,nom}, T_{nom}) = \frac{\alpha_{nom}}{\alpha_{nom}} \cdot \exp \left( \frac{E_{a}}{k} \left[ \frac{1}{T_{max,nom}} - \frac{1}{T_{nom}} \right] \right)
\]

\[
AF_{ov}(T_{max,nom}, T_{ov}) = \frac{\alpha_{ov}}{\alpha_{nom}} \cdot \exp \left( \frac{E_{a}}{k} \left[ \frac{1}{T_{max,nom}} - \frac{1}{T_{ov}} \right] \right) \cdot \left( \frac{T_{max,nom}}{T_{ov}} \right)^{2} \tag{6}
\]

We define Power-on-Hour (\( POH \)) as we can express \( POH_{nom} \) as [5]:

\[
POH_{nom} = AF_{nom}(T_{max,nom}, T_{nom}) \cdot MTTF_{nom} \cdot d_{c_{nom}} \tag{8}
\]

where \( T_{nom} \) is the peak temperature during the nominal operation and is determined from post-P&R total power, core and chip area at nominal signoff using transient analysis in the HotSpot tool [8] and \( d_{c_{nom}} \) is the duty cycle of the nominal operation in the entire lifetime. This is equivalent to using Black’s equation (3).

Similarly, \( POH_{ov} \) is expressed as [9]:

\[
POH_{ov} = AF_{ov}(T_{max,nom}, T_{ov}) \cdot MTTF_{nom} \cdot d_{c_{ov}} \tag{9}
\]

where \( T_{ov} \) is the peak temperature during the overdrive operation and is determined from post-P&R total power, core and chip area at overdrive signoff using transient analysis in the HotSpot tool [8] and \( d_{c_{ov}} \) is the duty cycle of the overdrive operation in the entire lifetime.

The interconnect will survive its lifetime as long as the following condition is met:

\[
POH_{ov} + POH_{nom} \leq t_{LIFE} \tag{10}
\]

If this condition is not met then the interconnect cannot sustain all the overdrive and nominal duration of operations, that is, \( V_{dd,ov} \) in the input is infeasible. We need to relax this value from the operating conditions and iterate until we find the one that satisfies Equation (10).

**Runtime monitoring post-signoff:** Another way this effect can be accounted is at runtime by monitoring remaining lifetime after each cycle of operation. Figure 1 shows the various modes of operation.

Figures 1(a) and (b) show the cases when the overdrive and normal modes of operation take place in succession with no intervening idle period. Figures 1(c) and (d) show the cases when the overdrive and normal modes of operation take place with intervening idle periods, \( t_{idle,1} \) and \( t_{idle,2} \). Figures 1(a) and (d) show that the overdrive mode happens before the nominal mode of operation. In such cases, the interconnect can operate for \( t_{nom} \) if the following condition is satisfied:

\[
t_{LIFE} \geq AF_{ov}(T_{max,nom}, T_{ov}) \cdot t_{ov} + AF_{nom}(T_{max,nom}, T_{nom}) \cdot t_{nom} \tag{11}
\]

Figures 1(b) and (b) show that the overdrive mode happens after the nominal mode of operation. In such cases, the interconnect can operate for \( t_{ov} \) if the following condition is satisfied:

\[
t_{LIFE} \geq AF_{nom}(T_{max,nom}, T_{nom}) \cdot t_{nom} + AF_{ov}(T_{max,nom}, T_{ov}) \cdot t_{ov} \tag{12}
\]

All other cases will be a combination of these and can be derived with these sets of equations. In the general case, lifetime can be expressed as:

\[
t_{LIFE} = t_{idle} + \sum_{i=0}^{M} t_{ov,i} + \sum_{j=0}^{N} t_{nom,j} \tag{13}
\]

where \( t_{idle} \) is the total duration when the interconnect is idle, that is, current is not flowing through it, \( t_{ov,i} \) is the duration of overdrive mode in the \( i \)th cycle, \( t_{nom,j} \) is the duration of nominal mode in the \( j \)th cycle, \( M \) is the maximum number of overdrive mode cycles and \( N \) is the maximum number of nominal mode cycles such that the lifetime conditions described by Equations (11) and (12) are met. Figure 2 shows this behavior.

\( M \) and \( N \) are to be determined based on the input parameters and will provide a bound on duration of overdrive and nominal modes of operations respectively. The actual duty cycle of these modes of operations will be given as:
\[ \text{dc}_{\text{nom}} = \frac{t_{\text{LIFE}}}{N} \sum_{j=0}^{N} t_{\text{nom},j} \quad (14) \]

\[ \text{dc}_{\text{ov}} = \frac{t_{\text{LIFE}}}{M} \sum_{i=0}^{M} t_{\text{ov},i} \quad (15) \]

Figure 2: Cycling of idle, overdrive and nominal modes of operation.

2 Transient Temperature Analysis

We use transient temperature analysis using Hotspot [8] to estimate the peak temperature at nominal and overdrive modes of operation depending on their duration. We choose our sampling interval as 10 µs and the base frequency of MPU as 1GHz. Figures 3(a) – (d) show that even though the duty cycle is same, the effective duration of operation affects peak temperature. The first cycle in these plots is used for the warm-up phase as required by the tool and the peak temperature for our experiments is noted from the second cycle. We observe from figures 3(c) and (d) that at 50% duty cycle, when the duration of operation of 100 µs of overdrive causes a peak temperature of about 53°C whereas duration of 1 ms causes the peak to be at about 60°C. The steady-state is reached approximately after 1 ms.

References


Figure 3: Peak temperature (°C) vs. duration of operation at the same duty cycle. Duty Cycle = 0.2 – (a) 100 µs (b) 1 ms. Duty Cycle = 0.5 – (c) 100 µs (d) 1 ms.


