What is a graph?

A (directed) graph $G$ is

- A nonempty **set of vertices** $V$, also called *nodes* and

- A **set of edges** $E$, each pointing from one (tail) vertex to another (head) vertex. (A directed edge is denoted with an “arrow”)

![Graph Image](image-url)
Variants of graphs

**Undirected graph**: no arrows on edges.

“If there’s an edge between v and w then there's an edge between w and v.” (More precisely: an edge $e_{vw}$ connects the unordered pair of vertices $\{v,w\}$.)

**Multigraph**: undirected graph that may have multiple edges between a pair of nodes. Such edges are sometimes called *parallel* edges.

**Simple graph**: undirected graph with no self-loops (edge from v to v) and no parallel edges.

**Mixed graph**: directed graph that may have multiple edges between a pair of nodes as well as self-loops.
Graphs are everywhere
Graphs are everywhere

The internet graph
Graphs are everywhere

Map coloring
Graphs are everywhere

Path planning for robots
Graphs are everywhere
Are these the same graph?

A. Yes: the set of vertices is the same.
B. Yes: we can rearrange the vertices so that the pictures look the same.
C. No: the pictures are different.
D. No: the left graph has a crossing and the right one doesn't.
E. None of the above.
Representing directed graphs

Diagrams with vertices and edges

How many vertices?
For each ordered pair of vertices \((v, w)\)
how many edges go from \(v\) to \(w\)?
Representing directed graphs

Diagrams with vertices and edges

How many vertices? \( n \)
For each ordered pair of vertices \((v, w)\) how many edges go from \( v \) to \( w \)?

How many ordered pairs of vertices are there?
A. \( n \)
B. \( 2n \)
C. \( n^2 \)
D. \( n(n-1)/2 \)
E. \( 2^n \)
Representing directed graphs

Diagrams with vertices and edges

How many vertices? $n$
For each ordered pair of vertices $(v, w)$ how many edges go from $v$ to $w$?

Need to store $n^2$ ints
Representing directed graphs

**Adjacency matrix** $n \times n$ matrix:
- entry in row $i$ and column $j$ is the number of edges from vertex $i$ to vertex $j$

Rosen p. 669
Representing directed graphs

**Adjacency matrix** $n \times n$ matrix:
entry in row $i$ and column $j$ is the number of edges from vertex $i$ to vertex $j$

What can you say about the adjacency matrix of a loopless graph?

A. It has all zeros.
B. All the elements below the diagonal are 1.
C. All the elements are even.
D. All the elements on the diagonal are 0.
E. None of the above.

Rosen p. 669
Representing directed graphs

**Adjacency matrix** $n \times n$ matrix:
- entry in row $i$ and column $j$ is the number of edges from vertex $i$ to vertex $j$

What can you say about the adjacency matrix of a graph with no parallel edges?

A. It has no zeros.
B. It is symmetric.
C. All the entries above the diagonal are 0.
D. All entries are either 0 or 1.
E. None of the above.

Rosen p. 669
**Representing directed graphs**

**Adjacency matrix** $n \times n$ matrix:
- entry in row $i$ and column $j$ is the number of edges from vertex $i$ to vertex $j$

What can you say about the adjacency matrix of an **undirected** graph?

A. It has no zeros.
B. It is symmetric.
C. All the entries above the diagonal are 0.
D. All entries are either 0 or 1.
E. None of the above.

*Rosen p. 669*
Representing undirected graphs

Simple undirected graph:
* Only need to store the adjacency matrix above diagonal.

What's the maximum number of edges a simple undirected graph with n vertices can have?

A. \( n^2 \)
B. \( n^2/2 \)
C. \( n(n-1)/2 \)
D. \( n(n+1)/2 \)
E. \( n \)
When is an adjacency matrix an **inefficient** way to store a graph?

When there is a **high density** of edges compared to number of vertices ???

When there is a **low density** of edges compared to number of vertices ???
Representing directed graphs

**Adjacency list** (list of lists):
for each vertex v, associate list of all *neighbors* of v.

Rosen p. 668
The **neighbors** of a vertex $v$ are all the vertices $w$ for which there is an edge whose endpoints are $v,w$.

If two vertices are neighbors then they are called **adjacent** to one another.
The degree of a vertex in an undirected graph is the total number of edges incident with it, except that a loop contributes twice.

What's the maximum degree of a vertex in this graph?

A. 0
B. 1
C. 2
D. 3
E. None of the above.
What's the degree of vertex 5?
A. 5
B. 3
C. 2
D. 1
E. None of the above.
What's the degree of vertex 0?
A. 5
B. 3
C. 2
D. 1
E. None of the above.
Handshakes

If there are \( n \) people in a room, and each shakes hands with \( d \) people, how many handshakes take place?

A. \( n \)
B. \( d \)
C. \( nd \)
D. \( (nd)/2 \)
E. None of the above.
Handshakes

If there are \( n \) people in a room, and each shakes hands with \( d \) people, how many handshakes take place?

A. \( n \)  
B. \( d \)  
C. \( nd \)  
D. \( (nd)/2 \)  
E. None of the above.

*Don't double-count each handshake!*
Handshakes "in" graphs

If a simple graph has **n vertices** and each vertex has **degree d**, how many edges are there?

$$2 \ |E| = n \times d$$
Handshakes “in” graphs

If any graph has \textbf{n vertices}, then

$$2 |E| = \text{sum of degrees of all vertices}$$
Handshakes "in" graphs

If any graph has \( n \) vertices, then

\[
2 |E| = \text{sum of degrees of all vertices}
\]

What can we conclude?
A. Every degree in the graph is even.
B. The number of edges is even.
C. The number of vertices with odd degree is even.
D. The number self loops is even.
E. None of the above.
Puzzles
Tartaglia's Pouring Problem

You can pour from one cup to another until the first is empty or the second is full.

Large cup: contains 8 ounces, can hold more.
Medium cup: is empty, has 5 ounce capacity.
Small cup: is empty, has 3 ounce capacity

Can we divide the coffee in half? How?

A. Yes  
B. No
Tartaglia's Pouring Problem

Large cup: contains 8 ounces, can hold more.  
Medium cup: is empty, has 5 ounce capacity.  
Small cup: is empty, has 3 ounce capacity

You can pour from one cup to another until the first is empty or the second is full.

**Can we divide the coffee in half? How?**

*Hint: configurations (l,m,s) code # ounces in each cup*

A. Yes
B. No
Tartaglia's Pouring Problem
Tartaglia's Pouring Problem

Rephrasing the problem:

Looking for path from (8,0,0) to (4,4,0)
Path

Sequence

\((v_0, e_1, v_1, e_2, v_2, \ldots, e_k, v_k)\)

describes a route through the graph from

**start vertex** \(v_0\)

to

**end vertex** \(v_k\)
Tartaglia's Pouring Problem

Rephrasing the problem:

(1) Is there a **path** from (8,0,0) to (4,4,0)?
(2) If so, what's the **best** path?

"Best" means "shortest length"
What's the shortest length of a path from (8,0,0) to (4,4,0) ?
A. 7
B. 8
C. 14
D. 16
E. None of the above.
Algorithmic questions related to paths

**Reachability**: // “decision”
Does there exist a path from vertex v to vertex w?

**Path**: // “construction”
Find a path from vertex v to vertex w.

**Distance**: // “optimization”
What’s the length of the shortest path from vertex v to vertex w?
Exam Announcements

Exam on Monday
Covers through Day 9
(no graphs)

Bring student ID.
One handwritten note sheet
(8.5” x 11”, both sides).
Look up seat assignment.
No calculators.
No blue books.

Review Session covering the Practice Midterm
Saturday 1-3pm
Sunday 12-2pm
(selected by Piazza vote)

Good luck!

Seating Chart to be posted on Website