CSE 21: Mathematics for Algorithms and Systems Analysis
Lists and Selection

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Agenda

1. Introduction
2. Sets and Lists
3. Counting Sequences
4. Permutations and Combinations

iClicker Frequency: BA
About Me

- David Lisuk
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  - Office Hours: Thursday 4 - 5 pm, Friday 2 - 3 pm EBU 3B B250A
  - Discussion Hours: Wednesday 4 and 5 pm
- First year masters student in machine learning
- Previously TAed for CSE 140 with Alex Orailoglu
- Worked for 3 years between undergrad and graduate school
How would you like me to primarily spend my time in discussion?
(a) Work though example problems
(b) Go over lecture material from a different perspective
(c) Work out solutions to past homework problems
(d) Something else (please talk to me after class or email me ideas)
(e) I am only here because it’s required, I won’t be paying attention.
What are Sets and Lists

A set is:
- An collection of *distinct* items
- A set is *unordered*
- Notated \{a, b, c\}

A list is:
- A *ordered* collection of elements
- Not necessarily distinct elements
- Notated \(b, d, a, d\) or \(bdad\)

Sets and lists can be described by their cardinality (number of elements)
For Example a \(k\)-list is a list with \(k\) elements and a \(n\)-set is a set with \(n\) elements
Set notation

- $x \in A$: $x$ is an element of the set $A$
- $B \subseteq A$: $B$ is a subset of $A$ (every element in $B$ is also in $A$)
- $B \subset A$: $B$ is a proper subset of $A$, in addition to being a subset of $A$ there is at least one element in $A$ that is not in $B$
- $\emptyset = \{\}$: the empty set
- $C = A \cap B$: $C$ is the intersection of $A$ and $B$
- $C = A \cup B$: $C$ is the union of $A$ and $B$
- $|A| = k$ means the cardinality of $A$ is $k$ ($A$ is a $k$-set or $k$-list)
- $A \setminus B$ is the set of all elements in $A$ but not $B$
- Given $A \subseteq \Omega$, $\overline{A} = \Omega \setminus A$, $\overline{A}$ is the complement of $A$
Theorem 1

**Theorem**

*There are* $n^k$ *possible ways to build a* $k$*-list using only elements of an* $n$*-set.*

**Example:** How many 2-lists are made from the set $\{x, y, z\}$?

- $n = |\{x, y, z\}| = 3$
- There are $3^2 = 9$ 2-lists from the set $\{x, y, z\}$

<table>
<thead>
<tr>
<th>$\ell_1 \setminus \ell_2$</th>
<th>$x$</th>
<th>$y$</th>
<th>$z$</th>
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<tbody>
<tr>
<td>$x$</td>
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<td>$z$</td>
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Theorem 2 - Rule of Product

If you make a sequence of \( k \) choices (elements in a list) with each choice \( i \) being among \( c_i \) options, the number of total choice sequences is:

\[
\prod_{i=1}^{k} c_i
\]
Theorem 2 - Rule of Product

iClicker Question (Frequency: BA)

When counting the number of odd numbers between 100 and 500, you make 3 choices. What is the value of $c_1$ (the left most digit), $c_2$ (center digit), and $c_3$ (rightmost digit)?

(a) $c_1$, $c_2$, $c_3 = 5, 10, 10$

(b) $c_1$, $c_2$, $c_3 = 4, 9, 10$

(c) $c_1$, $c_2$, $c_3 = 5, 10, 5$

(d) $c_1$, $c_2$, $c_3 = 4, 10, 5 \times \{1, 2, 3, 4\}, \{0 − 9\}, \{1, 3, 5, 7, 9\}$

(e) $c_1$, $c_2$, $c_3 = 4, 9, 5$
Theorem 3 - Rule of Sums

Theorem

If you can partition your sequences into disjoint subsets, the total count equals the sum of the count in each subset.

Example: Student ID numbers come in two forms: Awxyz and Uxyz with w,x,y,z being digits 0-9. How many total id numbers are there?

- We can partition the numbers into two disjoint subsets: those starting with A and those starting with U
- There are $10^4$ numbers starting with A
- There are $10^3$ numbers starting with U
- Thus there are $10^4 + 10^3 = 11000$ total id numbers
Often something will be very difficult to count but it’s complement is easy to count. The complement rule states ($A \subseteq \Omega$, $\Omega$ is the “universe”):

$$|A| = |\Omega| - |\bar{A}|$$

**Example:** How many 5 bit binary numbers have at least one 1?

- Use rule of sums? # with 1 one + # with 2 ones, ...
- Much easier is noting that only 00000 has no 1’s
- Thus there are $2^5 - 1$ 5 bit binary numbers with at least one 1
Lists without Repetition (Theorem 4)

Given some $n$-set we wish to construct a $k$-list (order matters) with no repetitions.

i.e.: $xy$ and $xz$ are ok, but $xx$ is not ok

- For the first element, we have $n$ choices
- For the second element we have $n - 1$ choices, and so on
- For the $k$ choice, we’d have $n - k + 1$ choices left

Theorem

The number of $k$-list with no repetitions from an $n$-set equals:

$$\prod_{i=0}^{k-1} (n - i) = \frac{n!}{(n - k)!}$$

These are the *permutations* of the set, can be written as $P(n,k)$ in WeBWorK
Given some $n$-set, we wish to construct a $k$-subset (order doesn’t matter).

i.e.: from \{x, y, z\}, \{x, y\}, \{x, z\}, \{y, z\} are all the 2-subsets

- Count the number of $k$-lists which can be formed by the given $n$-set
- Each of the final $k$-subsets occurs in $k!$ different orderings
- Divide the count of lists by $k!$ to get the number of subsets

Theorem

The number of $k$-subsets from an $n$-set equals:

$$\frac{n!}{(n-k)!k!}$$

These are the \textit{combinations} of the set, can be written as $C(n,k)$ in WeBWorK
Wrap Up Problems

iClicker Question (Frequency: BA)

Lets say you have a bag with 10 marbles. If we pick 2, how many unique pairs of marbles could you see?
(a) P(10,2)
(b) $10^2$
(c) C(10,2) *
iClicker Question (Frequency: BA)

Let's say you have 4 pairs of pants, 2 shirts, 1 tie, and 3 hats. If you must always wear pants and a shirt, how many ways can you dress yourself in the morning?

(a) 8
(b) 16
(c) 24
(d) 64 ✗
(e) 128