F1. Find the **error** in this false statement and correct it:

“Finding the chromatic number of a graph is NP-complete.”

F2. Find the **error** in this false statement and correct it:

“Solving the Traveling Salesman Problem NP-complete.”

F3. Is this problem “Is the number N=561 composite?” in NP?

F4. Count the number of proper 3-colorings of the graph $G$:

F5. Count the number of proper colorings of the graph $G$ above that uses at most one color. Count the number of proper colorings of the graph $G$ above that uses at most two colors. Count the number of proper colorings of the graph $G$ above that uses at most four colors.
F6. Given this set of colors \{1, 2, 3, 4, 5, 6\} greedily color the vertices of the graph using this ordering \(v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}\) of the vertices.

F7. Given this set of colors \{1, 2, 3, 4, 5, 6\} greedily color the vertices of the above graph using this ordering

\(v_1, v_2, v_3, v_5, v_7, v_9, v_{11}, v_4, v_6, v_8, v_{10}, v_{12}\) of the vertices.

F8. Explain the difference between greedy coloring and the chromatic number.
For F9 - F13 properly color the vertices of the graph using the minimum number of colors.

F9. The complete multipartite graph $K_{2,3,3,7}$.
F10. The Petersen graph
F11. The Grötzsch graph
F12. A random tree on 10 vertices.
F13. The graph G in question F6
F14. Are these two graphs isomorphic?

F15. In two line notation, list all isomorphisms from $G$ to $H$
F16. Are these two graphs isomorphic?

\begin{itemize}
  \item \textbf{G} \hfill \textbf{H}
  \begin{itemize}
    \item \text{v}_1 \quad \text{v}_2 \quad \text{v}_3 \\
    \item \text{v}_4 \quad \text{v}_5 \quad \text{v}_6 \\
    \item \text{v}_7 \quad \text{v}_8 \quad \text{v}_9 \\
    \item \text{v}_{10}
  \end{itemize}
  \begin{itemize}
    \item \text{u}_1 \quad \text{u}_2 \quad \text{u}_3 \\
    \item \text{u}_4 \quad \text{u}_5 \quad \text{u}_6 \\
    \item \text{u}_7 \quad \text{u}_8 \quad \text{u}_9 \\
    \item \text{u}_{10}
  \end{itemize}
\end{itemize}

F17. Explain why the YES / NO question is in \textbf{NP}: “Are the two graphs \textbf{G} and \textbf{H} isomorphic?”

F18. Verify that \( \begin{pmatrix} \text{v}_1 & \text{v}_2 & \text{v}_3 & \text{v}_4 & \text{v}_5 & \text{v}_6 & \text{v}_7 & \text{v}_8 & \text{v}_9 & \text{v}_{10} \\ \text{u}_1 & \text{u}_2 & \text{u}_3 & \text{u}_4 & \text{u}_5 & \text{u}_6 & \text{u}_7 & \text{u}_8 & \text{u}_9 & \text{u}_{10} \end{pmatrix} \) is an isomorphism between the two graphs \textbf{G} and \textbf{H} in F16.
F19. Given these five websites and links between them, how would you model this as a graph or a directed graph?

<table>
<thead>
<tr>
<th>Website</th>
<th>Links to websites</th>
</tr>
</thead>
<tbody>
<tr>
<td>vlsicad.ucsd.edu/courses/cse-s14/</td>
<td>Ø <a href="https://piazza.com/ucsd/spring2014/cse21/home">https://piazza.com/ucsd/spring2014/cse21/home</a></td>
</tr>
<tr>
<td></td>
<td>Ø webwork.cse.ucsd.edu/webwork2/CSE21_Spring2014</td>
</tr>
<tr>
<td><a href="http://www.soundcloud.com/ProfessorShadow">www.soundcloud.com/ProfessorShadow</a></td>
<td>Ø <a href="http://www.soundcloud.com/ProfessorShadow">www.soundcloud.com/ProfessorShadow</a></td>
</tr>
<tr>
<td><a href="http://www.instagram.com/ProfessorShadow">www.instagram.com/ProfessorShadow</a></td>
<td>Ø <a href="http://www.soundcloud.com/ProfessorShadow">www.soundcloud.com/ProfessorShadow</a></td>
</tr>
<tr>
<td>webwork.cse.ucsd.edu/webwork2/CSE21_Spring2014</td>
<td></td>
</tr>
<tr>
<td><a href="https://piazza.com/ucsd/spring2014/cse21/home">https://piazza.com/ucsd/spring2014/cse21/home</a></td>
<td>Ø vlsicad.ucsd.edu/courses/cse-s14/</td>
</tr>
<tr>
<td></td>
<td>Ø webwork.cse.ucsd.edu/webwork2/CSE21_Spring2014</td>
</tr>
</tbody>
</table>
F20. Using the greedy algorithm, find the minimum weight spanning tree.
F21. What is the total weight of the minimum spanning tree?
F22. Using the sorted edges method, find the length of the tour going through all six cities.
F23. Using the nearest neighbor method, find the length of the tour going through all six cities, starting and ending at A.
F24. \[ (1, 2, 3, 4) \circ (1, 2)(3, 5) \]^{-1}

F25. The cipher is given below in two line notation:

\[
\begin{pmatrix}
\end{pmatrix}
\]

Decrypt this message that was encrypted with the above cipher:
S U R J U D P P L Q J L Q W K H E D V H P H Q W V W L Q N V

F26. What is the probability that a random permutation on five letters is a derangement?
For F27–F29: Let $B$ be a random bipartite graph with independent sets of sizes $|X| = 3$ and $|Y| = 5$. Each edge $(x, y)$ of $B$ has probability $\frac{1}{5}$.

F27. Draw an example of a random bipartite graph with independent sets of sizes 3 and 5.

F28. What is the maximum number of edges in $B$?

F29. What is the probability that $B$ has 5 edges?

F30. What is the expected number of edges of $B$?
F31. How many solutions using positive integers are there to the equation 
\[ x_1 + x_2 + x_3 + x_4 + x_5 = 15? \]

F32. How many possible surjections are there from \( \{1, 2, 3, 4\} \mapsto \{A, B\}\)?

F33. How many possible injections are there from \( \{1, 2, 3, 4\} \mapsto \{A, B\}\)?

F34. How many possible injections are there from \( \{A, B\} \mapsto \{1, 2, 3, 4\}\)?

F35. \( N = 210 = 2 \times 3 \times 5 \times 7 \). How many ways can you write this as a product of non-negative integers? (Hint: The number 30 has five ways:
\[ 2 \times 3 \times 5 = 2 \times 15 = 6 \times 5 = 3 \times 10 = 30 \] )
F36. \( P(A) = 0.3, \ P(A \cup B) = 0.7, \) and \( P(B)^c = 0.6. \) Are \( A \) and \( B \) independent events?

F37. Among 60-year-old college professors, 10\% are smokers and 90\% are nonsmokers. The probability of a non-smoker dying in the next year is 0.005 and the probability of a smoker dying is 0.05. Given that one 60-year-old professor dies in the next year, what is the probability that the professor is a smoker?

F38. At a hospital’s emergency room, patients are classified and 20\% are critical, 30\% are serious and 50\% are stable. Of the critical patients, 30\% die; of the serious patients, 10\% die; and of the stable patients, 1\% die.

\[ \begin{align*} \text{a) What is the probability that a patient who dies was classified as critical?} \\
\text{b) What is the probability that a critical patient dies?} \end{align*} \]
F40. In Madison County, Alabama, a sample of 100 cases from 1960 are investigated, and the 100 defendants are interviewed as to their true innocence or guilt.

<table>
<thead>
<tr>
<th></th>
<th>$B_1$ Actually guilty</th>
<th>$B_2$ Actually innocent</th>
<th>totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>Jury finds guilty</td>
<td>35</td>
<td>45</td>
</tr>
<tr>
<td>$A_2$</td>
<td>Jury finds not guilty</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>totals</td>
<td></td>
<td>40</td>
<td>60</td>
</tr>
</tbody>
</table>

- (a) Assuming they answer honestly, what is the probability that a defendant who was actually innocent was found guilty by a jury?
- (b) What is the probability that a defendant that was found guilty by a jury was actually innocent?
- (c) Are the events $A_1$ and $B_2$ independent?
F41. The determinant of a matrix can be calculated by expansion along minors. The determinant \( \det(M) \) of the \( n \times n \) square matrix \( M \) can be recursively calculated as:
\[
\det(M) = m_{11} \cdot \det(M_{11}) + \cdots + (-1)^{1+j} \cdot m_{1j} \cdot \det(M_{1j}) + \cdots
\]
where \( M_{1j} \) is the submatrix formed by eliminating the first row and \( j \)th column of \( M \). Give an expression for \( R(n) \), the asymptotic number of multiplications and additions used to calculate the determinant of an \( n \times n \) matrix using the above recursion.

F42. \( \binom{N}{2} \) is \( O(?) \)

F43. \( !N \) (which counts the number of derangements on \( N \) letters) is \( O(?) \)

F44. Finding \( thee \) shortest tour through \( N \) cities (for the traveling salesman problem) is \( O(?) \)
F45. Roadtrip! You just joined a band and you are on a tour of the following six cities. Note that not all cities are connected by roads. Find the absolute shortest tour while visiting each city exactly once, starting and returning to San Diego.
F46. Street sweeping in South Park. If the street sweeper starts at the corner of Date and 28th Street, is it possible for the street sweeper to clean each side of the street and return to 28th and Date, without diving down any side of the street more than once?
F47. Mail Delivery in South Park. If the mail carrier starts at the corner of Date and 28th Street, is it possible for the mail carrier to deliver mail to each house and business and return to 28th and Date, without walking down a any sidewalk more than once?
F48. Map coloring. Color the map of the following countries so that no two countries that share a border get colored with the same color. Model this as a graph problem, solve the graph problem, then solve the original map coloring question.
F49. High speed network. In a new housing development the internet service provider needs to provide high speed access to each home \( \{A, B, C, D\} \) using the fewest underground cables. This network needs to be connected. Find the cheapest high speed network that connects all of the homes \( \{A, B, C, D\} \), using the fewest underground cables given the following underground cable costs:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>70</td>
<td>10</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>70</td>
<td>7</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>10</td>
<td>7</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>3</td>
<td>5</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>
F50. Find a way of communicating this graph to a computer, which cannot see images, only lists or arrays (i.e. matrices).
You are in a band on tour, with shows at: San Diego, CA, Topeka, KS, Washington D.C., Seattle, WA and Auburn, AL. Using the mileage between the cities, model this as a graph problem where you need to visit each city exactly once, starting and returning to your home in Auburn, AL.

<table>
<thead>
<tr>
<th></th>
<th>San Diego</th>
<th>Auburn</th>
<th>Seattle</th>
<th>Washington D.C.</th>
<th>Topeka</th>
</tr>
</thead>
<tbody>
<tr>
<td>San Diego</td>
<td>0</td>
<td>2059</td>
<td>1255</td>
<td>2614</td>
<td>1492</td>
</tr>
<tr>
<td>Auburn</td>
<td>2059</td>
<td>0</td>
<td>2631</td>
<td>748</td>
<td>860</td>
</tr>
<tr>
<td>Seattle</td>
<td>1255</td>
<td>2631</td>
<td>0</td>
<td>2764</td>
<td>1818</td>
</tr>
<tr>
<td>Washington D.C.</td>
<td>2614</td>
<td>748</td>
<td>2764</td>
<td>0</td>
<td>1117</td>
</tr>
<tr>
<td>Topeka</td>
<td>1492</td>
<td>860</td>
<td>1818</td>
<td>1117</td>
<td>0</td>
</tr>
</tbody>
</table>
F52. What is the length of the tour found by using the nearest neighbor method, starting at Auburn, AL and visiting all four other cities and then returning to Auburn, AL? (same problem as HW9, number 11a)

F53. What is the length of the tour found by using the sorted edges method, starting at Auburn, AL and visiting all four other cities and then returning to Auburn, AL? (same problem as HW9, number 11b)

F54. List all tours and calculate the cost for each tour. (See piazza post 678)

F55. What is the absolute shortest tour? What is the longest?

F56. Consider the problem: ”Is there a tour through all five cities that uses less than 7000 miles?” Is this problem in NP? Justify your answer.
F1. Find the **error** in this false statement and correct it: “Finding the chromatic number of a graph is NP-complete.”

This question is not even in NP, it is not a YES/NO question and verifying an answer could take a very long time. Suppose someone told you that the chromatic number of a graph with a thousand vertices and 250,000 edges was 49. How long would it take you to verify this claim? You would need to check that you could not color this huge graph with 48 colors, 47 colors, 46 colors, etc.

**The correct statement**

Finding the chromatic number of a graph is NP-hard. Asking if a graph can be colored with \( k \) colors is NP-complete (since a YES answer can be verified quickly, and this problem is reduced from 3-SAT which is also an NP-complete problem)
F2. Find the error in this false statement and correct it: “Solving the Traveling Salesman Problem NP-complete.”

This question is also not in NP, it is not a YES/NO question and verifying an answer could take a very long time. To verify that a tour is the shortest tour, you would need to check \((N-1)!/2\) tours.

**The correct statement**

Solving the Traveling Salesman Problem is NP-hard. Asking if there is a tour with total cost less than a fixed value \(k\) is NP-complete (since a YES answer can be verified quickly, and since you can reduce the decision version of TSP to the problem of Hamiltonian Cycle - an NP-complete problem). If you can find a tour using less than cost \(k\), you will have found a Hamiltonian cycle.
F3. Is this problem “Is the number $N=561$ composite?” in NP?

“Is the number $N=561$ composite?”

This problem is in NP. If someone tells you YES and presents you with $561 = 3 \times 11 \times 17$, you can verify this very quickly. Finding that factorization may take some time, but verifying the YES answer is fast.

Do you believe my YES answer? Try verifying it yourself, just multiply 33 times 17.
F4. Count the number of proper 3-colorings of the graph G:

There are 12 proper colorings of this graph. Three choices for $v_4$, two choices for $v_3$ once $v_4$ is colored, two choice for $v_2$ and only one choice for $v_1$. 
F5. Count the number of proper 1,2,4-colorings of the graph $G$:

- There are 0 proper colorings that use at most 1 color.
- There are 0 proper colorings that use at most 2 colors.
- Solution 1: With at most four colors, there are $4 \cdot 3 \cdot 3 \cdot 2 = 72$ proper colorings.
  - 4 choices for $v_4$
  - 3 choices for $v_3$
  - 3 choices for $v_2$
  - 2 choices for $v_1$
- Solution 2: There are $24 + \binom{4}{3} \cdot 12 = 72$ proper colorings that use at most 4 colors: using all four colors, there are $4! = 24$ valid colorings (every assignment of four colors is a proper coloring); and for every choice of three of the four colors, there are 12 valid 3-colorings.
F6. Given this set of colors \{1, 2, 3, 4, 5, 6\} greedily color the vertices of the graph using this ordering

\(v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}\) of the vertices.

Greedily coloring this graph with the ordering

\(v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}\) will require all 6 colors.
Given this set of colors \( \{1, 2, 3, 4, 5, 6\} \) greedily color the vertices of the graph using this ordering \( v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12} \) of the vertices.

Greedily coloring this graph with the ordering \( v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12} \) will require all 6 colors.
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Greedily coloring this graph with the ordering \(v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}\) will require all 6 colors.
F6. Given this set of colors \( \{1, 2, 3, 4, 5, 6\} \) greedily color the vertices of the graph using this ordering \( v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12} \) of the vertices.

Greedily coloring this graph with the ordering \( v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12} \) will require all 6 colors.
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Greedily coloring this graph with the ordering \( v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12} \) will require all 6 colors.
F6. Given this set of colors \( \{1, 2, 3, 4, 5, 6\} \) greedily color the vertices of the graph using this ordering \( \nu_1, \nu_2, \nu_3, \nu_4, \nu_5, \nu_6, \nu_7, \nu_8, \nu_9, \nu_{10}, \nu_{11}, \nu_{12} \) of the vertices.

![Graph Diagram]

Greedily coloring this graph with the ordering \( \nu_1, \nu_2, \nu_3, \nu_4, \nu_5, \nu_6, \nu_7, \nu_8, \nu_9, \nu_{10}, \nu_{11}, \nu_{12} \) will require all 6 colors.

![Colored Graph Diagram]
F6. Given this set of colors \{1, 2, 3, 4, 5, 6\} greedily color the vertices of the graph using this ordering \(v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}\) of the vertices.

Greedily coloring this graph with the ordering \(v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}\) will require all 6 colors.
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Greedily coloring this graph with the ordering \( v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12} \) will require all 6 colors.
F7. Given this set of colors \(\{1, 2, 3, 4, 5, 6\}\) greedily color the vertices of the graph using this ordering \(v_1, v_2, v_3, v_5, v_7, v_9, v_{11}, v_4, v_6, v_8, v_{10}, v_{12}\) of the vertices.

Greedily coloring this graph with the ordering \(v_1, v_2, v_3, v_5, v_7, v_9, v_{11}, v_4, v_6, v_8, v_{10}, v_{12}\) will require 3 colors.
F8. Explain the difference between greedy coloring and the chromatic number.

Finding the chromatic number is a hard problem (NP-hard and outside of NP). Greedy coloring is fast but it only gives a bound on the chromatic number. In practice for really really large graphs, you can choose a few orderings, quickly greedily color, then obtain a decent bound on the chromatic number.

If you choose an ordering and greedily color with $k$ colors, then $\chi(G) \leq k$
F9. Find the chromatic number of the complete multipartite graph $K_{2,3,3,7}$.

$$\chi(K_{2,3,3,7}) = 4$$
F10. Find the chromatic number of the Petersen Graph.

This proper 3-coloring shows $\chi(P) \leq 3$. Since there is a five cycle $\chi(P) \geq 3$.

$$\chi(P) \leq 3 \& \chi(P) \geq 3 \Rightarrow \chi(P) = 3$$
F11. Find the chromatic number of the Grötzcsh Graph.

These two different 4-colorings each show $\chi(G) \leq 4$. Since there is a five cycle, which is an odd cycle, then $\chi(G) \geq 3$.

On the left start with coloring the five cycle, then copy that coloring to the middle five vertices, you are forced to color the vertex in the center with a new color.

On the right color the outside cycle first, then color the vertex in the center with one of your three colors, say blue. Then the remaining five vertices must be a new color. No matter how hard you try there is no proper three coloring of this graph, four colors must be used.

$$\chi(G) = 4$$
F12. Find the chromatic number of a random tree on 10 vertices.

No matter which tree on 10 vertices you use, all trees (with at least one edge) have chromatic number 2, since all trees are bipartite.

\[ \chi(T_{10}) = 2 \]
F13. Find the chromatic number of a random tree on 10 vertices.

This graph is bipartite, therefore $\chi(G) = 2$
NO, $\chi(G) = 3$ and $\chi(H) = 2$, therefore they are not isomorphic. You can also say that $H$ is bipartite, but $G$ has an odd cycle (the 9-cycle around the outside $v_1, v_2, ..., v_9$).
F15. In two line notation, list all isomorphisms from $G$ to $H$

Let's first redraw the graph on the right.

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F15. In two line notation, list all isomorphisms from $G$ to $H$

Let’s first redraw the graph on the right.
F15. In two line notation, list all isomorphisms from $G$ to $H$

There are two symmetries...
F15. In two line notation, list all isomorphisms from $G$ to $H$

There are two symmetries...
F15. In two line notation, list all isomorphisms from $G$ to $H$

Do one, both or neither of the symmetries - this gives you four isomorphisms.

$$
\begin{pmatrix}
    v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\
    u_4 & u_2 & u_6 & u_1 & u_5 & u_3
\end{pmatrix}
\begin{pmatrix}
    v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\
    u_1 & u_5 & u_3 & u_4 & u_2 & u_6
\end{pmatrix}
$$

$$
\begin{pmatrix}
    v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\
    u_6 & u_2 & u_4 & u_3 & u_5 & u_1
\end{pmatrix}
\begin{pmatrix}
    v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\
    u_3 & u_5 & u_1 & u_6 & u_2 & u_4
\end{pmatrix}
$$
F16. Are these two graphs isomorphic?

YES, here is an isomorphism (verified in F18.)

\[
\begin{pmatrix}
v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} \\
u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & u_7 & u_8 & u_9 & u_{10}
\end{pmatrix}
\]
F17. Explain why the YES / NO question is in **NP**: “Are the two graphs G and H isomorphic?”

Look at the example of F16.

Here is a YES answer, a bijection which takes edges to edges and non-edges to non-edges - which can be verified to do so in polynomial time:

\[
\begin{pmatrix}
v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} \\
u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & u_7 & u_8 & u_9 & u_{10}
\end{pmatrix}
\]

Since a YES answer can be verified fast, this problem is in **NP**
F18. Verify that this is an isomorphism between $G$ and $H$.

$$\phi = \begin{pmatrix} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} \\ u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & u_7 & u_8 & u_9 & u_{10} \end{pmatrix}$$

All 15 edges in $G$ go to edges in $H$ and all non-edges in $G$ go to non-edges in $H$. For example, $(v_1, v_2)$ is an edge in $G$ and $(\phi(v_1), \phi(v_2)) = (u_1, u_2)$ is an edge in $H$.
F19. Given these five websites and links between them, how would you model this as a graph or a directed graph?
Using the greedy algorithm, find the minimum weight spanning tree.
F21. What is the total weight of the minimum spanning tree?

1 + 2 + 2 + 3 + 5 + 6 + 7 + 8 + 11 + 12 = 57
F22. Using the sorted edges method, find the length of the tour going through all six cities.

\[1 + 2 + 3 + 8 + 9 + 14 = 37\]
F23. Using the nearest neighbor method, find the length of the tour going through all six cities.
F23. Using the nearest neighbor method, find the length of the tour going through all six cities.
F23. Using the nearest neighbor method, find the length of the tour going through all six cities.

4 + 2 + 6 + ...
F23. Using the nearest neighbor method, find the length of the tour going through all six cities.

4 + 2 + 6 + 1...
F23. Using the nearest neighbor method, find the length of the tour going through all six cities.

\[ 4 + 2 + 6 + 1 + 8 + \ldots \]
F23. Using the nearest neighbor method, find the length of the tour going through all six cities.

\[4 + 2 + 6 + 1 + 8 + 9\]
F24. \([(1, 2, 3, 4) \circ (1, 2)(3, 5)]^{-1}\)

Let \(f = (1, 2, 3, 4), g = (1, 2)(3, 5)\). First we compute \(f \circ g = (1, 2, 3, 4) \circ (1, 2)(3, 5)\) (remember apply the map \(g\) first)

- \(3 \leftarrow f \ 2 \leftarrow g\ 1\)
- \(2 \leftarrow f \ 1 \leftarrow g\ 2\)
- \(5 \leftarrow f \ 5 \leftarrow g\ 3\)
- \(1 \leftarrow f \ 4 \leftarrow g\ 4\)
- \(4 \leftarrow f \ 3 \leftarrow g\ 5\)

In cycle notation this is \((1, 3, 5, 4)\), it's inverse is \((1, 3, 5, 4)^{-1} = (4, 5, 3, 1) = (1, 4, 5, 3)\)

Rubalcaba (rrrubalcaba@eng.ucsd.edu)
F25. Decrypt this message that was encrypted with the cipher:

\[
\begin{array}{cccccccccccccccc}
A & B & C & D & E & F & G & H & I & J & K & L & M & N & O & P \\
D & E & F & G & H & I & J & K & L & M & N & O & P & Q & R & S \\
T & U & V & W & X & Y & Z & A & B & C \\
\end{array}
\]

Decrypt this message that was encrypted with the above cipher:

SURJDPLQLQWKHEDVHPHQWVWLQNVP
F25. Decrypt this message that was encrypted with the cipher:

\[
\begin{array}{cccccccccccccccc}
A & B & C & D & E & F & G & H & I & J & K & L & M & N & O & P & Q \\
D & E & F & G & H & I & J & K & L & M & N & O & P & Q & R & S & T \\
U & V & W & X & Y & Z & A & B & C
\end{array}
\]

Decrypt this message that was encrypted with the above cipher:

SUR JUD PPL QJ LQ WKH EDV HPH QW VW LQ NV PR
F25. Decrypt this message that was encrypted with the cipher:

\[
\begin{array}{cccccccccccccccccccc}
\end{array}
\]

Decrypt this message that was encrypted with the above cipher:

SUJRUDPPLQQWKHEDVHPHQWVWLQNVP

PRO
F25. Decrypt this message that was encrypted with the cipher:

\[
\begin{array}{cccccccccccccccc}
\end{array}
\]

Decrypt this message that was encrypted with the above cipher:
S U R J U D P P L Q J L Q W K H E D V H P H Q W V W L Q N V

PROGRAMMING IN THE BASEMENT STINKS
F26. What is the probability that a random permutation on five letters is a derangement?

The function $!N$ counts the number of derangements on $N$ letters. You can compute $!5$ using the recursive formula (where $!0 = 1$ and $!1 = 0$).

$$!N = (N - 1)(!(N - 1) + !(N - 2))$$

$$!5 = (5 - 1)(!(5 - 1) + !(5 - 2)) = 4(!4 + !3)$$

$$!4 = (4 - 1)(!(4 - 1) + !(4 - 2)) = 3(!3 + !2)$$

$$!3 = (3 - 1)(!(3 - 1) + !(3 - 2)) = 2(!2 + !1)$$

$$!2 = (2 - 1)(!(2 - 1) + !(2 - 2)) = 1(!1 + !0)$$
F26. What is the probability that a random permutation on five letters is a derangement?

The function $!N$ counts the number of derangements on $N$ letters. You can compute $!5$ using the recursive formula (where $!0 = 1$ and $!1 = 0$).

$$!N = (N - 1)(!(N - 1) + !(N - 2))$$

$$!5 = (5 - 1)(!(5 - 1) + !(5 - 2)) = 4(4 + 3) =$$

$$!4 = (4 - 1)(!(4 - 1) + !(4 - 2)) = 3(3 + 2) = 3(3 + 1) =$$

$$!3 = (3 - 1)(!(3 - 1) + !(3 - 2)) = 2(2 + 1) = 2(1 + 0) = 2$$

$$!2 = (2 - 1)(!(2 - 1) + !(2 - 2)) = 1(1 + 0) = 1(0 + 1) = 1$$
F26. What is the probability that a random permutation on five letters is a derangement?

The function \( \!N \) counts the number of derangements on \( N \) letters. You can compute \( \!5 \) using the recursive formula (where \( \!0 = 1 \) and \( \!1 = 0 \)).

\[
\!N = (N - 1)(\!(N - 1)+\!(N - 2))
\]

\[
\!5 = (5 - 1)(\!(5 - 1)+\!(5 - 2)) = 4(\!4 + \!3) = 4(\!4 + 2) = 4 \cdot 6 = 24
\]
\[
\!4 = (4 - 1)(\!(4 - 1)+\!(4 - 2)) = 3(\!3+2) = 3(2 + 1) = 3 \cdot 3 = 9
\]
\[
\!3 = (3 - 1)(\!(3 - 1)+\!(3 - 2)) = 2(\!2+1) = 2(1 + 0) = 2
\]
\[
\!2 = (2 - 1)(\!(2 - 1)+\!(2 - 2)) = 1(\!1+0) = 1(0 + 1) = 1
\]
F26. What is the probability that a random permutation on five letters is a derangement?

The function \( N \) counts the number of derangements on \( N \) letters. You can compute \( !5 \) using the recursive formula (where \( !0 = 1 \) and \( !1 = 0 \)).

\[
!N = (N - 1)(!(N - 1) + !(N - 2))
\]

\[
!5 = (5 - 1)(!(5 - 1) + !(5 - 2)) = 4(!4 + !3) = 4(!4 + 2) = 4(9 + 2) = 44
\]

\[
!4 = (4 - 1)(!(4 - 1) + !(4 - 2)) = 3(!3 + !2) = 3(2 + 1) = 9
\]

\[
!3 = (3 - 1)(!(3 - 1) + !(3 - 2)) = 2(!2 + !1) = 2(1 + 0) = 2
\]

\[
!2 = (2 - 1)(!(2 - 1) + !(2 - 2)) = 1(!1 + !0) = 1(0 + 1) = 1
\]
F26. What is the probability that a random permutation on five letters is a derangement?

The function \( !N \) counts the number of derangements on \( N \) letters. You can compute \( !5 \) using the recursive formula (where \( !0 = 1 \) and \( !1 = 0 \)).

\[
!N = (N - 1)(!(N - 1) + !(N - 2))
\]

\[
!5 = (5 - 1)(!(5 - 1) + !(5 - 2)) = 4(4 + 3) = 4(7) = 28
\]

\[
!4 = (4 - 1)(!(4 - 1) + !(4 - 2)) = 3(3 + 2) = 3(5) = 15
\]

\[
!3 = (3 - 1)(!(3 - 1) + !(3 - 2)) = 2(2 + 1) = 2(3) = 6
\]

\[
!2 = (2 - 1)(!(2 - 1) + !(2 - 2)) = 1(1 + 0) = 1(1) = 1
\]

There are 5! permutations on five letters, so the probability that a random permutation on five letters is a derangement is

\[
\frac{!5}{5!} = \frac{28}{120} \approx 0.3667
\]
F27. Draw an example of a random bipartite graph with independent sets of sizes 3 and 5.

Let $B$ be a random bipartite graph with independent sets of sizes $|X| = 3$ and $|Y| = 5$. Each edge $(x, y)$ of $B$ has probability $\frac{1}{5}$.

Each edge has probability $1/5$ of appearing.
F27. Draw an example of a random bipartite graph with independent sets of sizes 3 and 5.

Let $B$ be a random bipartite graph with independent sets of sizes $|X| = 3$ and $|Y| = 5$. Each edge $(x, y)$ of $B$ has probability $\frac{1}{5}$. 

![Bipartite Graph Example]
F27. Draw an example of a random bipartite graph with independent sets of sizes 3 and 5.

Let $B$ be a random bipartite graph with independent sets of sizes $|X| = 3$ and $|Y| = 5$. Each edge $(x, y)$ of $B$ has probability $\frac{1}{5}$. 
F28. What is the maximum number of edges in $B$?

Let $B$ be a random bipartite graph with independent sets of sizes $|X| = 3$ and $|Y| = 5$. Each edge $(x, y)$ of $B$ has probability $\frac{1}{5}$.

The maximum number of edges for a bipartite graph with independent sets of sizes $|X|$ and $|Y|$ is $|X| \cdot |Y| = 3 \cdot 5 = 15$.
F29. What is the probability that $B$ has 5 edges?

Let $B$ be a random bipartite graph with independent sets of sizes $|X| = 3$ and $|Y| = 5$. Each edge $(x, y)$ of $B$ has probability $\frac{1}{5}$.

\[
\binom{15}{5} \left(\frac{1}{5}\right)^5 \left(\frac{4}{5}\right)^{15-5} \approx 0.1032
\]
Let $B$ be a random bipartite graph with independent sets of sizes $|X| = 3$ and $|Y| = 5$. Each edge $(x, y)$ of $B$ has probability $\frac{1}{5}$.

Edges are formed by independent coin tosses which are binomial distributed (with $n=15$ and $p=0.2$), so the expected value (where $X$ is the random variable # of edges).

\[
E(X) = 15 \cdot \frac{1}{5} = 3
\]

You can also use linearity of expectation.
F31. How many solutions using positive integers are there to the equation \( x_1 + x_2 + x_3 + x_4 + x_5 = 15 \)?

This is a stars and bars problem, with 15 stars and four bars, but with each variable starting with 1 star.

\[
\begin{array}{|c|c|c|c|c|}
\hline
x_1 & x_2 & x_3 & x_4 & x_5 \\
\hline
* & * & * & * & * \\
\hline
\end{array}
\]

Now we have 10 stars left: here is one possible solution

\[
\begin{array}{|c|c|c|c|c|}
\hline
x_1 & x_2 & x_3 & x_4 & x_5 \\
\hline
*** & *** & ** & * \\
\hline
\end{array}
\]

corresponding to the original problem:

\[
\begin{array}{|c|c|c|c|c|}
\hline
x_1 & x_2 & x_3 & x_4 & x_5 \\
\hline
*** & *** & ** & * \\
\hline
\end{array}
\]

So we have 10 stars and 5-1= 4 bars, this is \( C(10 + 4, 4) \) or

\[
\binom{\text{stars + bars}}{\text{bars}} = \binom{10 + 4}{4} = \binom{14}{4} = 1001
\]
F32. How many possible surjections are there from \( \{1, 2, 3, 4\} \mapsto \{A, B\} \)?

One solution is using Sterling numbers of the second kind.

\[
S(4, 2) \cdot 2! = 7 \cdot 2 = 14.
\]

For this example you can also easily count the number of functions that are *not* onto. For a function with codomain of size two to not be onto, all of the elements must be sent to \( A \) or \( B \) (the image should have only one element). There are only two such functions, either everything is sent to \( A \) or everything is sent to \( B \). There are \( 2^4 \) possible functions, so

\[
2^4 - 2 = 14 \text{ are surjective.}
\]
F33. How many possible injections are there from \( \{1, 2, 3, 4\} \mapsto \{A, B\} \)?

Four things cannot be mapped to two without one of \( A \) or \( B \) being hit more than once.

0
F34. How many possible injections are there from \{A, B\} \rightarrow \{1, 2, 3, 4\}?

There are 4 choices of where to send \(A\) and 3 choices where to send \(B\).

\[ P(4, 2) = 4 \cdot 3 = 12 \]
F35. \( N = 210 = 2 \times 3 \times 5 \times 7 \). How many ways can you write this as a product of non-negative integers?

(Hint: The number 30 has five ways: \( 2 \times 3 \times 5 = 2 \times 15 = 6 \times 5 = 3 \times 10 = 30 \))

The four factors can be partitioned in how many ways?

\[
N = 210 = 2 \times 3 \times 5 \times 7 = (2 \times 3) \times 5 \times 7 = \ldots
\]

The number of partitions of a set of four elements is the Bell number \( B(4) = S(4, 1) + S(4, 2) + S(4, 3) + S(4, 4) = 15. \)
F36. $P(A) = 0.3$, $P(A \cup B) = 0.7$, and $P(B)^c = 0.6$. Are $A$ and $B$ independent events?

Let’s see if $P(A \cap B) = P(A)P(B)$.

\[
P(A \cap B) = P(A) + P(B) - P(A \cup B)
\]

\[
P(A \cap B) = 0.3 + (1 - 0.6) - 0.7 = 0
\]

No. $A$ and $B$ are not independent events.
Among 60-year-old college professors, 10% are smokers and 90% are nonsmokers. The probability of a non-smoker dying in the next year is 0.005 and the probability of a smoker dying is 0.05. Given that one 60-year-old professor dies in the next year, what is the probability that the professor is a smoker?

Let \( S \) be the event that a 60 year old professor is a smoker and let \( D \) be the event that the 60 year old professor will die in the next year.

\[
P(S|D) = \frac{P(S \cap D)}{P(D)} = \frac{(0.1)(0.05)}{(0.1)(0.05) + (0.9)(0.005)} \approx 0.5263
\]

Figure 2: A tree diagram
At a hospital’s emergency room, patients are classified and 20% are critical, 30% are serious and 50% are stable. Of the critical patients, 30% die; of the serious patients, 10% die; and of the stable patients, 1% die.

(a) What is the probability that a patient who dies was classified as critical?

Let $C$ be the event that a patient is critical, $S$ that a patient is serious, $T$ that a patient is stable, and let $D$ be the event that a patient dies.

\[
P(C|D) = \frac{P(C \cap D)}{P(D)} = \frac{(0.2)(0.3)}{(0.2)(0.3) + (0.3)(0.1) + (0.5)(0.01)} \approx 0.6316
\]

(b) What is the probability that a critical patient dies? $P(D|C) = 0.3$
F40. “Is it just me or is there no F39?”

In Madison County, Alabama, a sample of 100 cases from 1990 are investigated, and the 100 defendants are interviewed as to their true innocence or guilt.

<table>
<thead>
<tr>
<th>A_1 Jury finds guilty</th>
<th>B_1 Actually guilty</th>
<th>B_2 Actually innocent</th>
<th>totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_1</td>
<td>35</td>
<td>45</td>
<td>80</td>
</tr>
<tr>
<td>A_2 Jury finds not guilty</td>
<td>5</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>totals</td>
<td>40</td>
<td>60</td>
<td>100</td>
</tr>
</tbody>
</table>

(a) Assuming they answer honestly, what is the probability that a defendant who was actually innocent was found guilty by a jury?

\[ P(A_1|B_2) = \frac{45}{60} = 0.75 \]

(b) What is the probability that a defendant that was found guilty by a jury was actually innocent?

\[ P(B_2|A_1) = \frac{45}{80} = 0.5625 \]

(c) Are the events A_1 and B_2 independent?

No, \[ P(A_1|B_2) = 0.75 \neq 0.8 = \frac{80}{100} = P(A_1) \]
F41. Blah blah blah, Matrix determinants, blah blah blah, Recursion!

\[ R(n) = n + n - 1 + n \cdot R(n - 1) = n \cdot R(n - 1) + 2n - 1, \text{ with } R(1) = 0 \]

\[ R(n) = O(n!) \]
F42. \( \binom{N}{2} \) is \( O(?) \)

\[
\binom{N}{2} = \frac{N(N - 1)}{2} = \frac{1}{2}N^2 - \frac{1}{2}N
\]

\[
\binom{N}{2} = O(N^2)
\]
F43. $!N$ (which counts the number of derangements on $N$ letters) is $O(\cdot)$

You can compute $!N$ using the recursive formula (where $!0 = 1$ and $!1 = 0$).

$$!N = (N - 1)(!(N - 1) + !(N - 2))$$

$$!N = O(N!)$$
F44. Finding *thee* shortest tour through \( N \) cities (for the traveling salesman problem) is \( O(?) \)

To find *thee* shortest tour of \( N \) cities, you would need to check all \((N - 1)!/2\) tours. For the homework problem you had with just 5 cities, this would be 12 tours to check, see the piazza post 678 for an example of finding thee shortest tour. (Solution forthcoming, but work on it now). This is \( O(N!) \)
F45. Roadtrip! You just joined a band and you are on a tour of the following six cities.

Note that not all cities are connected by roads. Find the absolute shortest tour while visiting each city exactly once, starting and returning to San Diego.

What happens if we add the edge Los Angeles to Yuma (take the road from LA to Yuma).
F45. Roadtrip! You just joined a band and you are on a tour of the following six cities.

If we add the edge Los Angeles to Yuma, how do we get to San Francisco? Remember the answer will be a cycle on six vertices.
F45. Roadtrip! You just joined a band and you are on a tour of the following six cities.

Since our answer will be a cycle on six vertices, let’s look at how many cycles on six vertices there are. Note all of them must use these three edges:
F45. Roadtrip! You just joined a band and you are on a tour of the following six cities.

Since our answer will be a cycle on six vertices, let’s look at how many cycles on six vertices there are:
F45. Roadtrip! You just joined a band and you are on a tour of the following six cities.

Since our answer will be a cycle on six vertices, let’s look at how many cycles on six vertices there are:
F45. Roadtrip! You just joined a band and you are on a tour of the following six cities.

This tour costs $9 + 2 + 4 + 5 + 7 + 8 = 35$
F45. Roadtrip! You just joined a band and you are on a tour of the following six cities.

This tour costs $3 + 4 + 5 + 7 + 6 + 9 = 34$
F45. Roadtrip! You just joined a band and you are on a tour of the following six cities.

This tour costs $3 + 4 + 5 + 7 + 6 + 9 = 34$

which is cheaper than the other tour, so San Diego to Los Angeles to San Francisco to Fresno to Yuma to Phoenix back to San Diego is the shortest tour through all six cities (note that the reverse tour has the same cost)!
F46. Street Sweeping in South Park.

If the street sweeper starts at the corner of Date and 28th Street, is it possible for the street sweeper to clean each side of the street and return to 28th and Date, without diving down any side of the street more than once?
F46. Street Sweeping in South Park.

If the street sweeper starts at the corner of Date and 28th Street, is it possible for the street sweeper to clean each side of the street and return to 28th and Date, without diving down any side of the street more than once?

Go east on Date St,
F46. Street Sweeping in South Park.

If the street sweeper starts at the corner of Date and 28th Street, is it possible for the street sweeper to clean each side of the street and return to 28th and Date, without diving down any side of the street more than once?

Go east on Date St, then turn right on Granada Ave,
F46. Street Sweeping in South Park.

If the street sweeper starts at the corner of Date and 28th Street, is it possible for the street sweeper to clean each side of the street and return to 28th and Date, without diving down any side of the street more than once?

Go east on Date St, then turn right on Granada Ave, make a U turn,
F46. Street Sweeping in South Park.

If the street sweeper starts at the corner of Date and 28th Street, is it possible for the street sweeper to clean each side of the street and return to 28th and Date, without diving down any side of the street more than once?

Go east on Date St, then turn right on Granada Ave, make a U turn, turn right on Date St.
F46. Street Sweeping in South Park.

If the street sweeper starts at the corner of Date and 28th Street, is it possible for the street sweeper to clean each side of the street and return to 28th and Date, without diving down any side of the street more than once?

Then turn right on 29th Street,
F46. Street Sweeping in South Park.

If the street sweeper starts at the corner of Date and 28th Street, is it possible for the street sweeper to clean each side of the street and return to 28th and Date, without diving down any side of the street more than once?

Make a U turn,
F46. Street Sweeping in South Park.

If the street sweeper starts at the corner of Date and 28th Street, is it possible for the street sweeper to clean each side of the street and return to 28th and Date, without diving down any side of the street more than once?

Turn Right on Date St.
If the street sweeper starts at the corner of Date and 28th Street, is it possible for the street sweeper to clean each side of the street and return to 28th and Date, without diving down any side of the street more than once?

Turn Right on Dale Street (not labeled on this map)
F46. Street Sweeping in South Park.

If the street sweeper starts at the corner of Date and 28th Street, is it possible for the street sweeper to clean each side of the street and return to 28th and Date, without diving down any side of the street more than once?

Make a U turn
F46. Street Sweeping in South Park.

If the street sweeper starts at the corner of Date and 28th Street, is it possible for the street sweeper to clean each side of the street and return to 28th and Date, without diving down any side of the street more than once?

Right on Date St
F46. Street Sweeping in South Park.

If the street sweeper starts at the corner of Date and 28th Street, is it possible for the street sweeper to clean each side of the street and return to 28th and Date, without diving down any side of the street more than once?

turn Right on 30th Street
F46. Street Sweeping in South Park.

If the street sweeper starts at the corner of Date and 28th Street, is it possible for the street sweeper to clean each side of the street and return to 28th and Date, without diving down any side of the street more than once?

Make a U turn,
If the street sweeper starts at the corner of Date and 28th Street, is it possible for the street sweeper to clean each side of the street and return to 28th and Date, without diving down any side of the street more than once?

Turn right on Dale, turn right on Fern, turn right on Ash St.
F46. Street Sweeping in South Park.

If the street sweeper starts at the corner of Date and 28th Street, is it possible for the street sweeper to clean each side of the street and return to 28th and Date, without diving down any side of the street more than once?

Turn right on 28th St.
F46. Street Sweeping in South Park.

If the street sweeper starts at the corner of Date and 28th Street, is it possible for the street sweeper to clean each side of the street and return to 28th and Date, without diving down any side of the street more than once?

Turn right on Beech Street
F46. Street Sweeping in South Park.

If the street sweeper starts at the corner of Date and 28th Street, is it possible for the street sweeper to clean each side of the street and return to 28th and Date, without diving down any side of the street more than once?

Make a U turn,
F46. Street Sweeping in South Park.

If the street sweeper starts at the corner of Date and 28th Street, is it possible for the street sweeper to clean each side of the street and return to 28th and Date, without diving down any side of the street more than once?

Turn right on 28th St, turn right on Cedar St,
F46. Street Sweeping in South Park. - Euler Tour

If the street sweeper starts at the corner of Date and 28th Street, is it possible for the street sweeper to clean each side of the street and return to 28th and Date, without diving down any side of the street more than once?

Make a U turn, turn right on 28th St, stop when you reach the corner of Date where you started.
If the mail carrier starts at the corner of Date and 28th Street, is it possible for the mail carrier to deliver mail to each house and business and return to 28th and Date, without walking down any sidewalk more than once?

The same solution (Euler tour) for the directed graph works for this undirected graph.
Color the map of the following countries so that no two countries that share a border get colored with the same color. Model this as a graph problem, solve the graph problem, then solve the original map coloring question.
F48. Map coloring. - Proper Coloring (with $\chi(G)$ colors)

Color the map of the following countries so that no two countries that share a border get colored with the same color. Model this as a graph problem, solve the graph problem, then solve the original map coloring question. There is a complete graph on four vertices. All countries must get a different color.
F48. Map coloring.- Proper Coloring (with $\chi(G)$ colors)

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- Bolivia
- Argentina
- Uruguay
- Brazil
- French Guiana
- Suriname
- Guyana
- Venezuela
- Colombia
- Equador
- Peru
- Chile
- Paraguay
- Uruguay
- Argentina

Sample Final Questions 132 / 162
Color the map of the following countries so that no two countries that share a border get colored with the same color. Model this as a graph problem, solve the graph problem, then solve the original map coloring question.
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![Map of South America with countries labeled](image)

![Graph representation of the countries](image)
In a new housing development the internet service provider needs to provide high speed access to each home \( \{A, B, C, D\} \) using the fewest underground cables. This network needs to be connected. Find the cheapest high speed network that connects all of the homes \( \{A, B, C, D\} \), using the fewest underground cable costs:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
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<th>C</th>
<th>D</th>
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<tbody>
<tr>
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<td>3</td>
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```
A --- 70 --- B
| 10  |  |  |
|-----|  |  |
|     |  |  |
| 3   | 5  | 2  |
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In a new housing development the internet service provider needs to provide high speed access to each home \( \{A, B, C, D\} \) using the fewest underground cables. This network needs to be connected. Find the cheapest high speed network that connects all of the homes \( \{A, B, C, D\} \), using the fewest underground cables given the following underground cable costs:

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![Graph](image-url)
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![Diagram showing the network connections with cable costs and selected connections highlighted.](image-url)
F50. Find a way of communicating this graph to a computer
F50. Find a way of communicating this graph to a computer

\[ G \]

\[ \begin{array}{c}
  v_1 \\
  v_2 \\
  v_3 \\
  v_4 \\
  v_5 \\
  v_6 \\
\end{array} \]

\[
\begin{pmatrix}
  v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\
  v_1 & 1 & & & & \\
  v_2 & 1 & 1 & & & \\
  v_3 & & 1 & & & \\
  v_4 & & & & & \\
  v_5 & & & & & \\
  v_6 & & & & & \\
\end{pmatrix}
\]
F50. Find a way of communicating this graph to a computer
F50. Find a way of communicating this graph to a computer

\[ G \]

\[ \begin{align*}
&\begin{array}{cccc}
  v_1 & v_2 & v_3 \\
  v_4 & v_5 & v_6
\end{array} \\
\end{align*} \]

\[ \begin{pmatrix}
0 & 1 & 0 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 1 & 0
\end{pmatrix} \]
You are in a band on tour with shows at: San Diego, CA, Topeka, KS, Washington D.C., Seattle, WA and Auburn, AL. Using the mileage between the cities, model this as a graph problem where you need to visit each city exactly once, starting and returning to your home in Auburn, AL.

<table>
<thead>
<tr>
<th></th>
<th>San Diego</th>
<th>Auburn</th>
<th>Seattle</th>
<th>Washington D.C.</th>
<th>Topeka</th>
</tr>
</thead>
<tbody>
<tr>
<td>San Diego</td>
<td>0</td>
<td>2059</td>
<td>1255</td>
<td>2614</td>
<td>1492</td>
</tr>
<tr>
<td>Auburn</td>
<td>2059</td>
<td>0</td>
<td>2631</td>
<td>748</td>
<td>860</td>
</tr>
<tr>
<td>Seattle</td>
<td>1255</td>
<td>2631</td>
<td>0</td>
<td>2764</td>
<td>1818</td>
</tr>
<tr>
<td>Washington D.C.</td>
<td>2614</td>
<td>748</td>
<td>2764</td>
<td>0</td>
<td>1117</td>
</tr>
<tr>
<td>Topeka</td>
<td>1492</td>
<td>860</td>
<td>1818</td>
<td>1117</td>
<td>0</td>
</tr>
</tbody>
</table>

Rubalcaba (rrrubalcaba@eng.ucsd.edu)
Using Nearest neighbor, starting at Auburn, AL (first visit Washington D.C., then Topeka, (not back to Auburn), then San Diego, then Seattle, then back to Auburn. Our total cost is

$$748 + 1117 + 1492 + 1255 + 2631 = 7243$$
Using sorted edges our tour cost is

\[ 748 + 860 + 1255 + 1492 + 2764 = 7119 \]

Note that 1117 is next after 760 and 860, but adding it would create a premature cycle! Next is 1255, 1492,...
F54. Traveling Salesman Problem - Finding the shortest path - only way is brute force through all possibilities

There are \((5 - 1)!/2 = 12\) possible tours, let's find them all and add up the total distances (cost). We will keep track and update the minimum value.
F54. Traveling Salesman Problem

![Graph showing the Traveling Salesman Problem with cities and distances]

\[748 + 860 + 1255 + 1492 + 2764 = 7119\]

Min = 7119  
Max = 7119
F54. Traveling Salesman Problem

\[ 748 + 2764 + 1818 + 1492 + 2059 = 8881 \quad \text{Min} = 7119 \quad \text{Max} = 8881 \]
F54. Traveling Salesman Problem

748 + 1117 + 1492 + 1255 + 2631 = 7243

Min = 7119
Max = 8881
F54. Traveling Salesman Problem

2631 + 2764 + 1117 + 1492 + 2059 = 10063

Min = 7119
Max = 10063
F54. Traveling Salesman Problem

2059 + 1255 + 2764 + 1117 + 860 = 8055

Min = 7119
Max = 10063
F54. Traveling Salesman Problem

\[
2059 + 748 + 1117 + 1818 + 1255 = 6997 \quad Min = 6997 \quad Max = 10063
\]
F54. Traveling Salesman Problem

748 + 2614 + 1255 + 1818 + 860 = 7259  \hspace{1cm} Min = 6997  \hspace{1cm} Max = 10063
F54. Traveling Salesman Problem

\[2059 + 2614 + 2764 + 1818 + 860 = 10115\]

\[\text{Min} = 6997\]

\[\text{Max} = 10115\]
F54. Traveling Salesman Problem

Washington D.C.

Seattle

San Diego

Topeka

Auburn

2764

1818

1255

1492

2614

1117

2631

860

10239

Min  =  6997  
Max  =  10239

2631 + 1818 + 1117 + 2614 + 2059 = 10239
F54. Traveling Salesman Problem

2631 + 2764 + 2614 + 1492 + 860 = 10311  \quad Min = 6997  \quad Max = 10311
2631 + 1255 + 2614 + 1117 + 860 = 8447  \quad Min = 6997  \quad Max = 10311
F54. Traveling Salesman Problem

2631 + 1818 + 1492 + 2614 + 748 = 9303

Min = 6997

Max = 10311
F54. Traveling Salesman Problem

Min = 6997 \quad Max = 10311
F55. Traveling Salesman Problem Find the shortest and longest tours

Min = 6997  Max = 10311

San Diego, Washington D.C., Seattle, Auburn, Topeka, San Diego for 10311 miles.
F55. Consider the problem: "Is there a tour through all five cities that uses less than 7000 miles?" Is this problem in NP?

This problem is in NP. It is a YES/NO question and YES claimed answers can be verified fast (poly time).

Here is a YES answer: San Diego, Auburn, Washington D.C., Topeka, Seattle, San Diego for 6997 miles, that can be verified in polynomial time.

Verification: $2059 + 748 + 1117 + 1818 + 1255 = 6997$ and all cities visited.

Note this not the same thing as asking for the best answer, if 7000 was changed to 10000, "Is there a tour through all five cities that uses less than 10000 miles?" there are many YES answers.