Pascal's Triangle
Probability It’s Friday > 0.5

- Bender & Williamson
  http://cseweb.ucsd.edu/~gill/BWLectSite/ (*including answers to problems in the text*)
- Study Sessions after class today and weeks 4 and 5
- Binomial Theorem (two examples)
- Venn Diagrams
- Functions (*chalkboard*)
- Permutations
- quiz
- field trip ”The Loft” after class
Study Sessions with Rob, Tutors, & TAs - all ages welcome

- **Week 3:** Friday (4/18) 2-4pm at The Loft (for HW3, and any general questions)
- **Week 4:** Friday (4/25) 2-4pm at The Loft (HW4, general questions)
- **Week 5:** Wed. (4/30) 2-4pm at The Loft (Midterm Review)
Midterm in Two Weeks...

GRAFFITI BEACH
Presents Resident DJ
Professor Shadow
Artist Showcase
ft. Eric Wixon & Nick McPherson
FRIDAY MAY 2ND 6 - 9 PM
de Méré initially wagered 24 rolls to get two six’s

<table>
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<th>$R$</th>
<th>$1 - \left( \frac{35}{36} \right)^R$</th>
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Pascal’s Triangle

Binomial identity:  *Pascal’s triangle*

\[
\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}
\]
Pascal’s Triangle

Proof 1: Using formula:

\[ LHS = \binom{n}{k} + \binom{n}{k+1} \]

\[ = \frac{n!}{k!(n-k)!} + \frac{n!}{(k+1)!(n-k-1)!} \]

\[ = \frac{n!(k+1)}{k!(n-k-1)!(n-k)(k+1)} + \frac{n!(n-k)}{(k+1)!(n-k-1)!(n-k)} \]

\[ = \frac{n!(k+1) + n!(n-k)}{(k+1)!(n-k)!} \]

\[ = \frac{n!(k+1 + n-k)}{(k+1)!(n-k)!} = \frac{n!(n+1)}{(k+1)!(n-k)!} = \binom{n+1}{k+1} \]
(x + y)^3 = (x + y)(x^2 + yx + xy + y^2)
= (x^3 + xyx + x^2y + xy^2) + (yx^2 + y^2x + yxy + y^3)
= (x^3 + xyx + x^2y + xy^2) + (yx^2 + y^2x + yxy + y^3)
= x^3 + xyx + x^2y + yx^2 + xy^2 + y^2x + yxy + y^3
= x^3 + 3x^2y + 3xy^2 + y^3

= \binom{3}{0}x^{3-0}y^0 + \binom{3}{1}x^{3-1}y^1 + \binom{3}{2}x^{3-2}y^2 + \binom{3}{3}x^{3-3}y^3
Binomial Theorem

\[(3a - 5b)^7\]

Use the theorem with \(x = 3a\) and \(y = -5b\)

\[(x + y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k\]

\[
= \binom{7}{0} (3a)^7 0 (-5b)^0 + \binom{7}{1} (3a)^7 1 (-5b)^1 + \cdots + \binom{7}{7} x^7 7 (-5b)^7 \\
3^7 a^7 - 5 \cdot 3^6 \cdot 7 a^6 b + \cdots
\]
Let $A$ and $B$ be events such that $P(A \cap B) = 1/4$, $P(A^C) = 1/3$, and $P(B) = 1/2$. What is $P(A \cup B)$?
Let $A$ and $B$ be events such that $P(A \cap B) = \frac{1}{4}$, $P(A^C) = \frac{1}{3}$, and $P(B) = \frac{1}{2}$. What is $P(A \cup B)$?
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$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \left(1 - \frac{2}{3}\right) + \frac{1}{2} - \frac{1}{4}$$
Functions

Let $f : A \rightarrow B$ be a function.

- If $\forall b \in B \ \exists^{\geq 1} a \in A ( f(a) = b )$ then $f$ is a **surjection** or **onto** function.
- If $\forall b \in B \ \exists^{\leq 1} a \in A ( f(a) = b )$ then $f$ is a **injection** or **one-to-one** function.
- If $f$ is both an injection and a surjection it is called a **bijection**.
- If $A = B$ and $f$ is a bijection then it’s called a **permutation** of $A$.

In this course, because we want to **count**, most of the sets that we’ll be considering are finite. As a consequence, we have many choices for how to represent functions.
Representing functions

1. Relations: set of pairs
2. Arrows: domain set on left, codomain set on right
   - Translate each of the function properties to properties of these pictures.
3. Two line notation: matrix-like, with domain elements on top row and codomain elements on bottom.
   - Translate each of the function properties to properties of these pictures. In particular, what happens if the function is not surjective?
4. Cycle notation: **use only if $f$ is a permutation**
Next Week

For week 4 we will talk more about functions and permutations. We will also discuss Random Graphs and Probability Distributions.
Quiz 3 (5 minute question preview)

1. If you toss a fair coin 5 times, what is the probability that you will get exactly three heads?

2. You roll a die three times. What is the probability that at least one 4 appears?

3. Given the set of poker hands $F =$ Flush (all suits the same), $SF =$ Straight Flush and $R =$ Royal Flush. We know that there are far more Flushes than Straight Flushes, and more Straight Flushes than Royal Flushes. $|F| > |SF| > |R|$. How do the three probabilities relate? $P(F) \ ? \ P(SF) \ ? \ P(R)$

4. $P(A) = 0.25$, $P(A \cup B) = 0.40$, $P(A \cap B) = 0.05$. Find $P(B)$

E.C. There are six ways of connecting one plugboard wire to a pair of letters on a mini Enigma plugboard using four letters $\{A, B, C, D\}$. If the one wire is randomly connected to a pair of letters, what is the probability that A is not sent to another letter?
Quiz Question 1

If you toss a fair coin 5 times, what is the probability that you will get exactly three heads?

A. $\frac{1}{32}$  
B. $\frac{3}{32}$  
C. $\frac{5}{32}$  
D. $\frac{8}{32}$  
E. $\frac{10}{32}$
Quiz Question 2

You roll a die three times. What is the probability that at least one 4 appears?

A. \( \frac{3}{6} \)

B. \( \left( \frac{1}{6} \right)^3 \)

C. \( \left( \frac{5}{6} \right)^3 \)

D. \( 1 - \left( \frac{1}{6} \right)^3 \)

E. \( 1 - \left( \frac{5}{6} \right)^3 \)
Quiz Question 3  

Given the set of poker hands $F =$ Flush (all suits the same), $SF =$ Straight Flush and $R =$ Royal Flush. We know that there are far more Flushes than Straight Flushes, and more Straight Flushes than Royal Flushes. 

$|F| > |SF| > |R|$. How do the three probabilities relate? 

$P(F) \ ? \ P(SF) \ ? \ P(R)$ 

A. $P(F) = P(SF) = P(R)$ 
B. $P(F) < P(SF) = P(R)$ 
C. $P(F) = P(SF) < P(R)$ 
D. $P(F) > P(SF) > P(R)$ 
E. $P(F) < P(SF) < P(R)$
Quiz Question 4

\( P(A) = 0.25, \quad P(A \cup B) = 0.40, \quad P(A \cap B) = 0.05. \) Find \( P(B) \)

A. 0.10  B. 0.20  C. 0.30  D. −0.10  E. 0.70
There are six ways of connecting one plugboard wire to a pair of letters on a mini Enigma plugboard using Four letters \{A, B, C, D\}. Here are all six permutations (listed in cycle notation: \((XY)\) means \(X\) goes to \(Y\) and \(Y\) goes to \(X\)): \{(AB), (CD), (AC), (BD), (BC), (AD)\}.

If the one wire is \textit{randomly} connected to a pair of letters, what is the probability that \(A\) is not sent to another letter (that is \(A\) is not connected to either end of the wire)?

\begin{align*}
A. & \quad 3! \\
B. & \quad \frac{1}{6} \\
C. & \quad \frac{2}{6} \\
D. & \quad \frac{3}{6} \\
E. & \quad \frac{4}{6}
\end{align*}
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