CSE21 - Math for Algorithm and Systems Analysis

http://vlsicad.ucsd.edu/courses/cse21-s14/
http://webwork.cse.ucsd.edu/webwork2/CSE21_Spring2014/
Homework 3 available!
https://piazza.com/ucsd/spring2014/cse21/home

Is it your unbirthday?
Cloudy with a Chance of Madvillain

- Introduction to Probability (Motivation)
  - Predicting the future
    - Failure analysis
  - Model(l)ing
- Poker probabilities
- Coin Tosses, Dice Rolls
- Birthday Paradox
  - Hash Collisions
- Further Reading (optional)
Tossin Coins

Given one fair coin, the likelihood of Heads or Tails is equally likely.

- \( P(\text{heads}) = P(\text{tails}) = \frac{1}{2} = 0.5 \)
Tossin Coins

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- $P(\text{heads}) = P(\text{tails}) = \frac{1}{2} = 0.5$

Given two fair coins, what is the probability of getting two heads?
Tossin Coins

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\[
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\]

Given two fair coins, what is the probability of getting two heads?

Let’s count! What are the possible outcomes?
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Given two fair coins, what is the probability of getting two heads? Let’s count! What are the possible outcomes?

\[ \{HH, HT, TH, TT\} \]
Tossing Coins

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Given two fair coins, what is the probability of getting two heads? Let’s count! What are the possible outcomes?

\[ \{HH, HT, TH, TT\} \]

\[ P(\text{heads, heads}) = \frac{1}{4} \]
Tossin Coins

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- \( P(\text{heads}) = P(\text{tails}) = \frac{1}{2} = 0.5 \)

Given two fair coins, what is the probability of getting two heads? Let’s count! What are the possible outcomes?

\[ \{HH, HT, TH, TT\} \]

\[ P(\text{heads, heads}) = \frac{1}{4} \]

Given three fair coins, what is the probability of getting three tails?
Tossin Coins

Given one fair coin, the likelihood of Heads or Tails is equally likely.

- \( P(\text{heads}) = P(\text{tails}) = \frac{1}{2} = 0.5 \)

Given two fair coins, what is the probability of getting two heads?
Let’s count! What are the possible outcomes?

\[
\{HH, HT, TH, TT\}
\]

\( P(\text{heads, heads}) = \frac{1}{4} \)

Given three fair coins, what is the probability of getting three tails?

\[
\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}
\]
Tossin Coins

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Given two fair coins, what is the probability of getting two heads?
Let’s count! What are the possible outcomes?

\[ \{HH, HT, TH, TT\} \]

\[ P(\text{heads, heads}) = \frac{1}{4} \]

Given three fair coins, what is the probability of getting three tails?

\[ \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\} \]

\[ P(\text{tails, tails, tails}) = \frac{1}{8} \]
Tossing Four Coins

Let's start with all possibilities for tossing two coins again:

H  H
H  T
T  H
T  T
Tossing Four Coins

Let's start with all possibilities for tossing three coins again (repeat the outcomes for two coins twice)

\[
\begin{array}{ccc}
H & H & H \\
H & H & T \\
H & T & H \\
H & T & T \\
T & H & H \\
T & H & T \\
T & T & H \\
T & T & T \\
\end{array}
\]
Tossing Four Coins

Let's start with all possibilities for tossing three coins again:

- H H H
- H H T
- H T H
- H T T
- T H H
- T H T
- T T H
- T T T
Tossing Four Coins

Repeat the outcomes for three coins twice once for the fourth coin heads and once for the fourth coin tails.

- H H H H T H H H H
- H H H T T H H H T
- H H T H T H H T H
- H H T T T H T T T T
- H T H H T T T H H
- H T H T T T H T T
- H T T H T T T T H
- H T T T T T T T T
What is probability of getting three heads when you toss a fair coin four times?
What is probability of getting three heads when you toss a fair coin four times?

\[
P(3 \text{ heads}) = \frac{4}{16} = \frac{1}{4} = 0.25
\]

We could have also counted the number of ways to have three heads in four tosses by \( \binom{4}{3} = \binom{4}{1} = 4 \).
Happy Unbirthday