Homework 2 available!

https://piazza.com/ucsd/spring2014/cse21/home
Outline for the next two lectures

- Counting Loop Iterations
- Poker Hands, Dice rolls, other Random Events
- Traveling Salesman Problem
- Permutations
- The Enigma Plugboard
Counting loop iterations

What is the value of $k$ after the following code has been executed?

$$
k := 0
$$

$$
\text{for } i_1 = 1 \text{ to } n_1
  \text{ for } i_2 = 1 \text{ to } n_2
    \text{ for } i_3 = 1 \text{ to } n_3
      k := k + 1
$$

A. 0
B. $n_1 + n_2 + n_3$
C. $n_1n_2n_3$
D. $n_1^2n_2^2n_3^2$
E. $n(n - 1)/2$
Counting loop iterations

What is the value of $k$ after the following code has been executed?

```plaintext
k := 0
for $i_1 = 1$ to $n_1$
    $k := k + 1$
for $i_2 = 1$ to $n_2$
    $k := k + 1$
for $i_3 = 1$ to $n_3$
    $k := k + 1$
```

A. 0
B. $n_1 + n_2 + n_3$
C. $n_1 n_2 n_3$
D. $n_1^2 n_2^2 n_3^2$
E. $n(n - 1)/2$
What is the value of k after the following code has been executed?

```
k := 0
for i_1 = 1 to n
    for i_2 = i_1 + 1 to n
        k := k + 1
```

A. 0  
B. \( \binom{n}{2} \)  
C. \( i_1(i_2 + 1) \)  
D. \( n^2 \)  
E. \( n(n - 1)/2 \)  
F. 37
# Standard Deck of Cards

## The Deck

- 13 Ranks \{2, 3, 4, 5, \ldots, K, A\}
- 4 Suits \{hearts, diamonds, clubs, spades\}

<table>
<thead>
<tr>
<th>2♥</th>
<th>3♥</th>
<th>4♥</th>
<th>5♥</th>
<th>6♥</th>
<th>7♥</th>
<th>8♥</th>
<th>9♥</th>
<th>10♥</th>
<th>J♥</th>
<th>Q♥</th>
<th>K♥</th>
<th>A♥</th>
</tr>
</thead>
<tbody>
<tr>
<td>2♦</td>
<td>3♦</td>
<td>4♦</td>
<td>5♦</td>
<td>6♦</td>
<td>7♦</td>
<td>8♦</td>
<td>9♦</td>
<td>10♦</td>
<td>J♦</td>
<td>Q♦</td>
<td>K♦</td>
<td>A♦</td>
</tr>
<tr>
<td>2♣</td>
<td>3♣</td>
<td>4♣</td>
<td>5♣</td>
<td>6♣</td>
<td>7♣</td>
<td>8♣</td>
<td>9♣</td>
<td>10♣</td>
<td>J♣</td>
<td>Q♣</td>
<td>K♣</td>
<td>A♣</td>
</tr>
<tr>
<td>2♠</td>
<td>3♠</td>
<td>4♠</td>
<td>5♠</td>
<td>6♠</td>
<td>7♠</td>
<td>8♠</td>
<td>9♠</td>
<td>10♠</td>
<td>J♠</td>
<td>Q♠</td>
<td>K♠</td>
<td>A♠</td>
</tr>
</tbody>
</table>
## Poker Hands

### (i) nothing:

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3♠</td>
<td>7♥</td>
<td>8♥</td>
<td>6♠</td>
<td>J♣</td>
<td></td>
</tr>
</tbody>
</table>

### (ii) one pair:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5♠</td>
<td>5♦</td>
<td>K♥</td>
<td>Q♥</td>
</tr>
</tbody>
</table>

### (iii) two pair:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>8♦</td>
<td>8♠</td>
<td>6♣</td>
<td>6♥</td>
</tr>
</tbody>
</table>

### (iv) three of a kind:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10♣</td>
<td>10♦</td>
<td>10♥</td>
<td>K♣</td>
</tr>
</tbody>
</table>

### (v) straight (not straight flush, not royal flush):

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>7♠</td>
<td>6♦</td>
<td>5♥</td>
<td>4♠</td>
</tr>
</tbody>
</table>

### (vi) flush (not straight flush, not royal flush):

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>8♣</td>
<td>7♣</td>
<td>5♣</td>
<td>4♣</td>
</tr>
</tbody>
</table>

### (vii) full house:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>9♠</td>
<td>9♣</td>
<td>9♥</td>
<td>4♦</td>
</tr>
</tbody>
</table>

### (viii) four of a kind:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2♠</td>
<td>2♣</td>
<td>2♥</td>
<td>2♦</td>
</tr>
</tbody>
</table>

### (ix) straight flush (not royal flush):

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5♠</td>
<td>4♠</td>
<td>3♠</td>
<td>2♠</td>
</tr>
</tbody>
</table>

### (x) royal flush:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A♥</td>
<td>K♥</td>
<td>Q♥</td>
<td>J♥</td>
</tr>
</tbody>
</table>
Count the number of Royal Flushes

Royal Flushes

- 4 Suits to choose \{ hearts, diamonds, clubs, spades \}

![Royal Flushes Diagram]
4 Suits to choose from \{ hearts, diamonds, clubs, spades \}
### Straight Flushes

- 4 Suits to choose from \{ \text{hearts, diamonds, clubs, spades} \}

<table>
<thead>
<tr>
<th>♠️</th>
<th>2♠️</th>
<th>3♠️</th>
<th>4♠️</th>
<th>5♠️</th>
</tr>
</thead>
<tbody>
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<td>5♠️</td>
<td>6♠️</td>
</tr>
<tr>
<td>3♠️</td>
<td>4♠️</td>
<td>5♠️</td>
<td>6♠️</td>
<td>7♠️</td>
</tr>
<tr>
<td>4♠️</td>
<td>5♠️</td>
<td>6♠️</td>
<td>7♠️</td>
<td>8♠️</td>
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<td>5♠️</td>
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<td>7♠️</td>
<td>8♠️</td>
<td>9♠️</td>
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<tr>
<td>6♠️</td>
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<td>9♠️</td>
<td>10♠️</td>
</tr>
<tr>
<td>7♠️</td>
<td>8♠️</td>
<td>9♠️</td>
<td>10♠️</td>
<td>J♠️</td>
</tr>
<tr>
<td>8♠️</td>
<td>9♠️</td>
<td>10♠️</td>
<td>J♠️</td>
<td>Q♠️</td>
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<tr>
<td>9♠️</td>
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<td>Q♠️</td>
<td>K♠️</td>
</tr>
<tr>
<td>10♠️</td>
<td>J♠️</td>
<td>Q♠️</td>
<td>K♠️</td>
<td>A♠️</td>
</tr>
</tbody>
</table>
Straight Flushes

- 4 Suits to choose from \{ hearts, diamonds, clubs, spades \}
Straight Flushes

- 4 Suits to choose from \{ hearts, diamonds, clubs, spades \}
Traveling Salesman Problem & Permutations
Let’s look at a smaller example, where our alphabet is only 4 letters \( \{A, B, C, D\} \). How many ways are there to connect one plugboard wire pair?
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- \( \binom{4}{2} = 6 \) ways to pick the pair of letters that are swapped

Let’s list them all in cycle notation:

\[ \{(AB)(C)(D), \]
Let’s look at a smaller example, where our alphabet is only 4 letters \( \{A, B, C, D\} \). How many ways are there to connect one plugboard wire pair?

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Let’s list them all in cycle notation:

\[
\{(AB)(C)(D)\}, \quad (CD),
\]
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Let’s list them all in cycle notation:

\[
\left\{ (AB)(C)(D), \ (CD), \ (AC), \ (BD), \right\}
\]
Let’s look at a smaller example, where our alphabet is only 4 letters \{A, B, C, D\}. How many ways are there to connect one plugboard wire pair?

- \(_\binom{4}{2} = 6\) ways to pick the pair of letters that are swapped.

Let’s list them all in cycle notation:

\[ \{(AB)(C)(D), \ (CD), \ (AC), \ (BD), \ (BC), \]
Let’s look at a smaller example, where our alphabet is only 4 letters \{A, B, C, D\}. How many ways are there to connect one plugboard wire pair?

\[
\binom{4}{2} = 6 \text{ ways to pick the pair of letters that are swapped}
\]

Let’s list them all in cycle notation:

\[
\{(AB)(C)(D), (CD), (AC), (BD), (BC), (AD)\}
\]

\((AB)(C)(D)\) Represents the permutation \((A \ B \ C \ D)\)
Our alphabet is only 4 letters \( \{A, B, C, D\} \). How many ways are there to connect \textit{two} plugboard wires?
Our alphabet is only 4 letters \( \{A, B, C, D\} \). How many ways are there to connect two plugboard wires?

- \( \binom{4}{2} = 6 \) ways to pick the pair of letters for the first wire
Our alphabet is only 4 letters \( \{A, B, C, D\} \). How many ways are there to connect two plugboard wires?

- \( \binom{4}{2} = 6 \) ways to pick the pair of letters for the first wire
- \( \binom{2}{2} = 1 \) way to pick the second wire
- The answer is not 6 \( \cdot \) 1.
Our alphabet is only 4 letters \( \{A, B, C, D\} \). How many ways are there to connect \textit{two} plugboard wires?

- \( \binom{4}{2} = 6 \) ways to pick the pair of letters for the first wire
- \( \binom{2}{2} = 1 \) way to pick the second wire

The answer is not 6 \( \cdot \) 1. It’s \( 6/2! = 3 \).

Let’s list them all: \( (AB)(CD) \),
Our alphabet is only 4 letters \( \{A, B, C, D\} \). How many ways are there to connect \textit{two} plugboard wires?

- \( \binom{4}{2} = 6 \) ways to pick the pair of letters for the first wire
- \( \binom{2}{2} = 1 \) way to pick the second wire

The answer is not 6 \cdot 1. It’s \( \frac{6}{2!} = 3 \)

Let’s list them all: \( (AB)(CD), \quad (AC)(BD), \)
Our alphabet is only 4 letters \{A, B, C, D\}. How many ways are there to connect \textit{two} plugboard wires?

- \(\binom{4}{2} = 6\) ways to pick the pair of letters for the first wire
- \(\binom{2}{2} = 1\) way to pick the second wire

The answer is not \(6 \cdot 1\). It’s \(6/2! = 3\)

Let’s list them all: \{(AB)(CD), (AC)(BD), (AD)(BC)\}

\[
\begin{pmatrix}
A & B & C & D \\
B & A & D & C
\end{pmatrix},
\begin{pmatrix}
A & B & C & D \\
C & D & A & B
\end{pmatrix},
\begin{pmatrix}
A & B & C & D \\
D & C & B & A
\end{pmatrix}
\]