HW8 available, take home quiz due Wed 5/21

Be vewy vewy quiet, I’m hunting wabbits...
Office Hours and Notes

Office Hours for Week 8

- Monday (5/19) 6-8pm with Amer in the CSE basement, maybe somewhere afters.
- Wednesday (5/21) 2-3pm at The Loft (with tutors, some TAs)
- Wednesday (5/21) 8-10pm at Homeplate (with tutors)
  - Make sure you hand in your take home quiz to me by 10pm at Homeplate, or turn it in during Wednesday’s class.

Other Notes

- Applications for tutors now available for CSE 20 Summer Session I
  https://academicaffairs.ucsd.edu/Modules/ASES/Apply.aspx?cid=875
- Programming Competition http://wic.ucsd.edu/competition.html
- Volunteer to help - free pizza! #freefood
- I’m on travel this Thursday - next Tuesday. I’ll still help out on Piazza from a different timezone. Still having class on Friday.
Topics for the week

- Recursion
  - Induction
  - Fibonacci numbers
  - The mating habits of *Oryctolagus cuniculus* (European rabbits).
  - Addition chains
  - Towers of Hanoi
  - Matrix determinants

- Complexity
  - Asymptotics
  - Big $O$ notation $O(n^2)$
  - Big Omega $\Omega(n^2)$
  - Big Theta $\Theta(n^2)$

- Complexity Zoo (maybe next week along with Graphs)
  - $P$
  - $NP$
  - $NP$ – complete
  - $NP$ – hard
  - $co$ – $NP$
Begin with one male and one female rabbit.
Rabbits can mate at the age of one month, so by the end of the second month, each female can produce another pair of rabbits.
The rabbits never die.
The female produces one male and one female every month.
The Mating Habits of *Oryctolagus cuniculus*

- Begin with one male and one female rabbit.
- Rabbits can mate at the age of one month, so by the end of the second month, each female can produce another pair of rabbits.
- The rabbits never die.
- The female produces one male and one female every month.

- Begin with one pair of new born rabbits. (1)
- At the end of the first month, still only one pair exists. (1)
- At the end of the second month, the female has produced a second pair, so two pairs exist. (2)
- At the end of the third month, the original female has produced another pair, and now three pairs exist. (3)
- At the end of the fourth month, the original female has produced yet another pair, and the female born two months earlier has produced her first pair, making a total of five pairs. (5)
Count the Rabbits

- 1, 1, 2, 3, 5, 8, 13,
- 1, 1, 2, 3, 5, 8, 13, 21 ...
- Notice a pattern?
Count the Rabbits

- $1, 1, 2, 3, 5, 8, 13,$
- $1, 1, 2, 3, 5, 8, 13, 21 \ldots$

**Notice a pattern?** (Note that all of the following are correct)

$$F_1 = 1, \quad F_2 = 1$$

A. $F_n = F_{n-1} + F_{n-2}$  
B. $F_{n+2} = F_{n+1} + F_n$  
C. $F_n = F_{n+2} - F_{n+1}$
A classic example of a recursive procedure is the function used to calculate the factorial of a natural number:

\[
\text{fact}(n) = \begin{cases} 
1 & \text{if } n = 0 \\
 n \cdot \text{fact}(n - 1) & \text{if } n > 0 
\end{cases}
\]

**Pseudocode (recursive):**

```plaintext
function factorial is:
input: integer n such that n >= 0
output: [n × (n-1) × (n-2) × ... × 1]

1. if n is 0, return 1
2. otherwise, return [ n × factorial(n-1) ]

end factorial
```
The function can also be written as a recurrence relation:

\[ b_n = nb_{n-1} \]
\[ b_0 = 1 \]

This evaluation of the recurrence relation demonstrates the computation that would be performed in evaluating the pseudocode above:

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### Computing the recurrence relation for \( n = 4 \):

\[
\begin{align*}
b_4 &= 4 \times b_3 \\
&= 4 \times (3 \times b_2) \\
&= 4 \times (3 \times (2 \times b_1)) \\
&= 4 \times (3 \times (2 \times (1 \times b_0))) \\
&= 4 \times (3 \times (2 \times (1 \times 1))) \\
&= 4 \times (3 \times (2 \times 1)) \\
&= 4 \times (3 \times 2) \\
&= 4 \times 6 \\
&= 24
\end{align*}
\]
Interactive puzzle:
http://www.mathsisfun.com/games/towerofhanoi.html

MIT video (see 2:00-19, in particular recursion is discussed with Dr. 4 starting at 13:00)
Towers of Hanoi (small cases)

- $T_1 = 1$ Move one disc from tower 1 to tower 3.
- $T_2 = 3$ Move small disc to tower 2, large disc to tower 3, small disc to tower 3.
- $T_3 = 7$

How do we find $T_4$?

Note: $T_3 = 2T_2 + 1$ (solve $T_2$ twice and move the largest disc once)

$T_4 = 2T_3 + 1$ (solve $T_2$ twice and move the largest disc once)

$$T_n = 2T_{n-1} + 1$$
Towers of Hanoi (pattern)

- $T_1 = 1$ Move one disc from tower 1 to tower 3.
- $T_2 = 3$ Move small disc to tower 2, large disc to tower 3, small disc to tower 3.
- $T_3 = 7 = 2^3 - 1$
- $T_4 = 15 = 2^4 - 1$

Possible pattern $T_n = 2^n - 1$
Towers of Hanoi (prove pattern by induction)

Claim: \( T_n = 2^n - 1 \)

Proof: (mathematical induction)

- **Base case**: \( T_0 = 0 = 2^0 - 1 \) holds.
- **Inductive Hypothesis**: \( T_k = 2^k - 1 \) for all non negative integers \( k \)
- **Inductive Step**: \( T_n = 2T_{n-1} + 1 = 2(2^{n-1} - 1) + 1 = 2^n - 1 \)
A brief introduction to the complexity zoo

We will talk about this in much more detail later. Today I only want to introduce a few basic fundamental facts about key complexity classes. There are 496 classes currently listed on the complexity zoo website. The following diagram only shows four: $P$, $NP$, $NP$-complete, and $NP$-hard.