HW7 available, take home quiz due next Wed 5/21

Long Live Wu-Tang Wednesday
Office Hours, and Notes

- Wednesday (5/14) Take home quiz
- **Wednesday (5/14) 5:30-7:00 EBU3B 4122**
- **Wednesday (5/14) 7:15-9:30pm Homeplate**
- **Friday (5/16) 12-12:45 EBU3B 4122**
- Friday (5/16) Sun God Festival!!
- Sunday (5/18) Homework HW7 due Sunday by 11:59pm
- Week 8 On official travel 5/22-5/27, Andrew guest lecturer Friday 5/23

Other Notes

- Dine with your professor - FREE food
  - https://students.ucsd.edu/academics/advising/academic-success/dine-with-a-prof.html
- New mixes on www.soundcloud.com/ProfessorShadow
Conditional Probability

Let $U$ be sample space with probability function $P$. If $A, B \subseteq U$ are events, then the conditional probability of $B$ given $A$ is

$$P(B|A) = \begin{cases} \frac{P(A \cap B)}{P(A)} & \text{if } P(A) \neq 0 \\ \text{undefined} & \text{if } P(A) = 0 \end{cases}$$

Properties of conditional probability

- $P(B|U) = P(B)$
- Conditioned on everything, we don’t get any new knowledge.
- $A$ and $B$ independent if and only if $P(B|A) = P(B)$.
- Intuition: independence means conditioning on new information doesn’t change probability.
- By definition of independence, $A$ and $B$ are independent if and only if $P(B|A)P(A) = P(A \cap B) = P(A)P(B)$ so divide out by $P(A)$. 
Bayes’ Theorem

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Bayes’ Theorem

\[
P(A|B) = \frac{P(B|A)P(A)}{P(B)}
\]

By definition, can write \( P(A \cap B) \) in two ways.
\[
P(B|A)P(A) = P(A \cap B) = P(A|B)P(B).
\]
Independence

\[ P(A) = 0.2 \quad P(B) = 0.5 \quad P(A \cup B) = 0.6 \]

Then \(0.6 = 0.2 + 0.5 - P(A \cap B), \) thus \(P(A \cap B) = 0.1\)

a. Find \(P(A|B)\)

b. Find \(P(B|A)\)

c. Are the events \(A\) and \(B\) independent?
Independence

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a. Find \( P(A|B) = \frac{0.1}{0.5} = 0.2 \)

b. Find \( P(B|A) = \frac{0.1}{0.2} = 0.5 \)

c. Are the events \( A \) and \( B \) independent? Yes, \( P(A|B) \neq P(A) \)
Monty Hall

Contestant chooses: Goat 1, Goat 2, Car

Host shows: Goat 2, Goat 1, Goat 2, Goat 1

Contestant switches? Y, N, Y, N, Y, N

Contestant gets: C, G1, C, G2, G2, C, C

Switch
- Car: \( \frac{2}{3} \)
- Goat 1: \( \frac{1}{6} \)
- Goat 2: \( \frac{1}{6} \)

No switch
- Car: \( \frac{1}{3} \)
- Goat 1: \( \frac{1}{3} \)
- Goat 2: \( \frac{1}{3} \)
At a hospital’s emergency room, patients are classified and 20% are critical, 30% are serious and 50% are stable. Of the critical patients, 30% die; of the serious patients, 10% die; and of the stable patients, 1% die.

(a) What is the probability that a patient who dies was classified as critical?

Let $C$ be the event that a patient is critical, $S$ that a patient is serious, $T$ that a patient is stable, and let $D$ be the event that a patient dies.

How do we answer (a)? - Dead dude that was classified as critical?

A. $P(C \cap D)$    B. $P(D|C)$    C. $P(C|D)$    D. $P(C \cup D)$    E. ??
Protect Ya Neck

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How do we answer (b) The probability that a critical patient dies?

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\[
P(C|D) = \frac{P(C \cap D)}{P(D)} = \frac{(0.2)(0.3)}{(0.2)(0.3) + (0.3)(0.1) + (0.5)(0.01)} \approx 0.6316
\]

(b) What is the probability that a critical patient dies? $P(D|C) = 0.3$
Let $A$ be the event of rolling a 7. Let $B$ be the event of rolling at least one “2.”

Find $P(A|B)$, the probability that you roll a 7 given that you roll at least one 2.

\[
P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{2/36}{11/36} = \frac{2}{11}
\]
Without factoring P or Q, perform probabilistic tests to help decide if they are composites or probable primes. You may use http://www.akalin.cx/intro-primality-testing or write code to calculate the following:

\[ P = 294571791067375389885907239089503408618560001 \]

1. Take the numerical part of your PID (a12345678 becomes \( a = 12345678 \)).
2. Calculate \( t = a^{P-1} \mod P \).
3. Question 2 was one randomized Fermat Primality test on \( P \). What can you conclude from your result?

\[ Q = 108628836495712919311107552952534229085134089 \]

4. Calculate \( r = a^{Q-1} \mod Q \). (use \( a \) as in part 1)
5. Question 4 was one randomized Fermat Primality test on \( Q \). What can you conclude from your result?