Midterm Friday
One 8.5 x 11 sheet (both sides) of *handwritten* notes, no electronics

Scattered, Smothered, and Covered
Office Hours / Study Sessions this week

- Wed. (4/30) 2-4:00pm at The Loft (Midterm Q&A w/ TAs, tutors)
- Wed. (4/30) 6-7:00pm at EBU3B 4122
- Thurs. (5/1) 3-4pm EBU3B 4122
- Thurs. (5/1) 7-9pm CENTER 119 (Midterm Q&A)
- Thurs. (5/1) 9-11pm Porters Pub - (tentative)
- Friday (5/2) 12-12:40 EBU3B 4122
Sample Midterm Questions

1. List all derangements of the four element set \{1, 2, 3, 4\} in cycle notation.

2. What is the probability that a random permutation on four letters is a derangement?

3. Find \(( (2, 3, 4, 5) \circ (1, 3, 5) \circ (1, 2)(3, 5) )^{-1}\)

4. Given an alphabet of just 6 letters, \{A, B, C, D, E, F\} how many plugboard wires would give the most possible arrangements?

5. How many ways are there to connect two plugboard wires with a restricted alphabet of five letters? How does this relate to Sterling Numbers of the second kind (explain the connection or lack of connection)?

6. How many ways are there to solve \(w+x+y+z=10\), if each of \(w,x,y,z\) are \textit{positive} integers?

7. What is the probability of getting a full house dealt from standard deck of 52 cards?

8. How many partitions of a set of five labeled elements \(\{A, B, C, D, E\}\) have two unlabeled subsets?

9. In a class with 22 students, what is the probability that at least two students share a birthday?

10. In a class with 22 students, what is the probability that \textit{no} students share a birthday?

11. How many injective functions are there from \(D = \{1, 2, 3\}\) to \(C = \{a, b, g, y, z\}\)?

12. How many surjective functions are there from \(D = \{1, 2, 3, 4\}\) to \(C = \{\text{red, blue}\}\)?

13. Encrypt this message with the Casear cipher: MIDTERM

14. What is the probability that a random graph \(G(15, 0.2)\) has 15 edges? What is the expected number of edges?

15. List all partitions of five labeled elements that have three unlabeled subsets of size 3,1, and 1. Is this \(S(5, 3)\)?

16. Calculate the number of ways to partition a set of five labeled objects into three \textit{labeled} subsets of size 3,1,1 in two different ways, one with multinomial coefficients and one way with binomial coefficients.
Additional Sample Midterm Questions

17. Given \( P(A) = \frac{3}{10} \), \( P(A \cap B)^c = \frac{9}{10} \), \( P(A \cup B) = \frac{7}{10} \). Find \( P(B) \).

18. There are 10 light bulbs, 2 of which are defective. If you randomly select 3 bulbs, what is the probability that you will get two working light bulbs and one defective bulb?

19. A local UCSD organization held a raffle to raise money. They sold $1 raffle tickets for an iPad valued at $300. If they sold 1000 tickets, what is the expected value of one raffle ticket?

20. Let \( D \) be a directed graph on \( N \) vertices, how many directed edges are possible?

21. *Deep Thoughts:* Is the composition of two permutations on \( n \) labeled objects always another permutation on the same \( n \) labeled objects? Can two different letters (such as \( E \) and \( Z \)) be mapped to the same letter (such as \( M \)) after composing two permutations on the set \( \{A, B, ..., Z\} \)?

22. Count number of permutations on six letters that fix one letter, fix two letters.

23. Hash browns at Waffle House always come "scattered (on the grill)" but you can also ask for them to be: smothered, covered, chunked, diced, peppered, capped, topped, or country. You can ask for at most one of any of these extras, for example the typical Waffle House order would ask for hash browns "Scattered, Smothered, and Covered." How many ways can you order hash browns at Waffle House?

24. Playlists: Given a hard drive with 1000 songs, how many ways can you make a playlist of 20 songs (repeats allowed)? How many ways can you make a playlist of 20 songs with no repeats?

25. Essay Question (see last slide)
M17. Given $P(A) = \frac{3}{10}$, $P(A \cap B)^c = \frac{9}{10}$, $P(A \cup B) = \frac{7}{10}$. Find $P(B)$ (Note diagram is not drawn to scale).

\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]

A. 0.3    B. 0.4    C. 0.5    D. 0.6    E. 0.7
Midterm Practice Question 17

**M17.** Given \( P(A) = \frac{3}{10}, \) \( P(A \cap B)^c = \frac{9}{10}, \) \( P(A \cup B) = \frac{7}{10}. \) Find \( P(B) \) 

*(Note diagram is not drawn to scale)*

\[
P(A \cup B) = P(A) + P(B) - P(A \cap B)
\]

\[
0.7 = 0.3 + P(B) - (1 - 0.9)
\]

\[
0.7 = 0.3 + P(B) - 0.1
\]

\[
0.7 - 0.3 + 0.1 = P(B)
\]

A. 0.3  
B. 0.4  
C. 0.5  
D. 0.6  
E. 0.7
M18. There are 10 light bulbs, 2 of which are defective. If you randomly select 3 bulbs, what is the probability that you will get two working light bulbs and one defective bulb?
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\[
\frac{\binom{2}{1} \cdot \binom{8}{2}}{\binom{10}{3}} = \frac{28 \cdot 2}{120} \approx 0.4667
\]
M19. A local UCSD organization held a raffle to raise money. They sold $1 raffle tickets for an iPad valued at $300. If they sold 1000 tickets, what is the expected value of one raffle ticket?

Let $X$ be the amount won or lost. If you don’t win the iPad you lose one dollar, or $X_1 = -1$. If you win the iPad you have won the equivalent of $300 minus the one dollar you spent on the ticket.
Midterm Practice Question 19

**M19.** A local UCSD organization held a raffle to raise money. They sold $1 raffle tickets for an iPad valued at $300. If they sold 1000 tickets, what is the expected value of one raffle ticket?

Let $X$ be the amount won or lost. If you don’t win the iPad you lose one dollar, or $X_1 = -1$. If you win the iPad you have won the equivalent of $300 minus the one dollar you spent on the ticket.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$f(X)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>300 - 1</td>
<td>$\frac{1}{1000} = 0.001$</td>
</tr>
<tr>
<td>-1</td>
<td>$\frac{999}{1000} = 0.999$</td>
</tr>
</tbody>
</table>
M19. A local UCSD organization held a raffle to raise money. They sold $1 raffle tickets for an iPad valued at $300. If they sold 1000 tickets, what is the expected value of one raffle ticket?

Let \( X \) be the amount won or lost. If you don’t win the iPad you lose one dollar, or \( X_1 = -1 \). If you win the iPad you have won the equivalent of $300 minus the one dollar you spent on the ticket.

\[
\begin{array}{c|c}
X & f(X) \\
\hline
300 - 1 & \frac{1}{1000} = 0.001 \\
-1 & \frac{999}{1000} = 0.999 \\
\end{array}
\]

\[
E(X) = \sum X_i \cdot P(X_i)
\]

where the sum is taken over all possible values of \( X_i \).

\[
E(X) = 299 \cdot (0.001) + (-1)(0.999) = -0.70
\]
M20. Let $D$ be a directed graph on $N$ vertices, how many directed edges are possible?

Each pair of vertices has two directed edges possible, one from $v_a$ to $v_b$ and one from $v_b$ to $v_a$.

A. $C(N,2)$  B. $2 \cdot C(N,2)$  C. $2 \cdot P(N,2)$  D. $P(N,2)$
M20. Let D be a directed graph on N vertices, how many directed edges are possible?

Each pair of vertices has two directed edges possible, one from $v_a$ to $v_b$ and one from $v_b$ to $v_a$. So twice as many directed edges as an undirected graph (undirected graphs can have at most $\binom{N}{2}$ undirected edges).

Or, we are counting how many ordered pairs there are from N objects.

A. $C(N, 2)$  
B. $2 \cdot C(N, 2)$  
C. $2 \cdot P(N, 2)$  
D. $P(N, 2)$
M21. Deep Thoughts: Is the composition of two permutations on $n$ labeled objects always another permutation on the same $n$ labeled objects? Can two different letters (such as $E$ and $Z$) be mapped to the same letter (such as $M$) after composing two permutations on the set $\{A, B, ..., Z\}$?
M22. Count number of permutations on six letters \{Q, W, E, R, T, Y\} that fix the letter Q.

\[
\begin{pmatrix}
Q & W & E & R & T & Y \\
Q & * & * & * & * & *
\end{pmatrix}
\]

A. \(6!\)  
B. \(6! - 1!\)  
C. \(5!\)  
D. \(\frac{6!}{1!}\)  
E. ??
M22. Count number of permutations on six letters \{ Q, W, E, R, T, Y \} that fix the letter \( Q \).

\[
\begin{pmatrix}
Q & W & E & R & T & Y \\
Q & * & * & * & * & *
\end{pmatrix}
\]

A. 6!  B. 6! – 1!  C. 5!  D. \( \frac{6!}{1!} \)  E. ??

Note this is the same as counting all permutations on five letters \{ W, E, R, T, Y \}
M22. Count number of permutations on six letters \{Q, W, E, R, T, Y\} that fix the letters \(T\) and \(Y\).

\[
\begin{pmatrix}
Q & W & E & R & T & Y \\
* & * & * & * & T & Y \\
\end{pmatrix}
\]

A. 6!  
B. 6! − 2!  
C. 4!  
D. \(\frac{6!}{2!}\)  
E. ??
M22. Count number of permutations on six letters \( \{Q, W, E, R, T, Y\} \) that fix the letters \( T \) and \( Y \).

\[
\begin{pmatrix}
Q & W & E & R & T & Y \\
* & * & * & * & T & Y
\end{pmatrix}
\]

A. 6!  
B. 6! - 2!  
C. 4!  
D. \( \frac{6!}{2!} \)  
E. ??

Note this is the same as counting all permutations on *four* letters \( \{Q, W, E, R\} \)
Midterm Practice Question 23

M23a. Hash browns at waffle house always come ”scattered” (spread on the grill), but you can also ask for them to be:

- ”smothered” (with onions)
- ”covered” (with cheese)
- ”chunked” (with diced ham)
- ”diced” (with diced tomatoes)
- ”peppered” (with jalapeo peppers)
- ”capped” (with mushrooms)
- ”topped” (with Bert’s chili)
- ”country” (with sausage gravy)

How many ways are there to ask for a single plate of hash browns at Waffle House (at most one of each extra, no repeats)?

A. $8!$  
B. $2^8$  
C. $\binom{2 + 8 - 1}{8 - 1}$  
D. $S(8, 2)$  
E. $\binom{8}{2}$
M23a. Hash browns at waffle house always come "scattered" (spread on the grill), but you can also ask for them to be:

- "smothered" (with onions)
- "covered" (with cheese)
- "chunked" (with diced ham)
- "diced" (with diced tomatoes)
- "peppered" (with jalapeño peppers)
- "capped" (with mushrooms)
- "topped" (with Bert's chili)
- "country" (with sausage gravy)

How many ways are there to ask for a single plate of hash browns at Waffle House (at most one of each extra, no repeats)? Each extra has 2 possibilities: either add the extra or don’t, so there are ___ possible ways.

A. 8!  
B. $2^8$  
C. $\binom{2 + 8 - 1}{8 - 1}$  
D. $S(8, 2)$  
E. $\binom{8}{2}$
M23b. Hash browns at waffle house always come ”scattered” (spread on the grill), but you can also ask for them to be:

- ”smothered” (with onions)
- ”covered” (with cheese)
- ”chunked” (with diced ham)
- ”diced” (with diced tomatoes)
- ”peppered” (with jalapeo peppers)
- ”capped” (with mushrooms)
- ”topped” (with Bert’s chili)
- ”country” (with sausage gravy)

How many ways are there to ask for a single plate of hash browns at Waffle House with exactly two extras (no repeats)?

A. 8!  B. $2^8$  C. $\binom{2 + 8 - 1}{8 - 1}$  D. $S(8, 2)$  E. $\binom{8}{2}$
M23b. Hash browns at waffle house always come "scattered" (spread on the grill), but you can also ask for them to be:

- "smothered" (with onions)
- "covered" (with cheese)
- "chunked" (with diced ham)
- "diced" (with diced tomatoes)
- "peppered" (with jalapeo peppers)
- "capped" (with mushrooms)
- "topped" (with Bert's chili)
- "country" (with sausage gravy)

How many ways are there to ask for a single plate of hash browns at Waffle House with exactly two extras (no repeats)? Here you are choosing 2 things from 8, without regard to order.

A. $8!$
B. $2^8$
C. $\binom{2 + 8 - 1}{8 - 1}$
D. $S(8, 2)$
E. $\binom{8}{2}$
Midterm Practice Question 23

**M23c.** Hash browns at Waffle House always come "scattered" (spread on the grill), but you can also ask for them to be:

- "smothered" (with onions)
- "covered" (with cheese)
- "chunked" (with diced ham)
- "diced" (with diced tomatoes)
- "peppered" (with jalapeño peppers)
- "capped" (with mushrooms)
- "topped" (with Bert’s chili)
- "country" (with sausage gravy)

How many ways are there to ask for a single plate of hash browns at Waffle House with exactly two extras (repeats allowed: i.e. you can ask for covered twice)?

- **A.** $8!$
- **B.** $2^8$
- **C.** $\left(\frac{2 + 8 - 1}{8 - 1}\right)$
- **D.** $S(8, 2)$
- **E.** $\binom{8}{2}$
M23c. Hash browns at waffle house always come ”scattered” (spread on the grill), but you can also ask for them to be:

- ”smothered” (with onions)
- ”covered” (with cheese)
- ”chunked” (with diced ham)
- ”diced” (with diced tomatoes)
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- ”capped” (with mushrooms)
- ”topped” (with Bert’s chili)
- ”country” (with sausage gravy)

How many ways are there to ask for a single plate of hash browns at Waffle House with exactly two extras (repeats allowed: i.e. you can ask for covered twice)? *Stars and bars.* 7 bars, 2 stars. You can also do \( 8 \times 7 / 2! + \) the 8 repeats, *but this method becomes tedious for larger values of ”2”.*

A. 8! B. \( 2^8 \) C. \( \binom{2 + 8 - 1}{8 - 1} \) D. \( S(8, 2) \) E. \( \binom{8}{2} \)
**Midterm Practice Question 24**

**M24a.** Playlists: Given a hard drive with 1000 songs, how many ways can you make a playlist of 20 songs (repeats allowed)?

- **A.** \( \binom{1000}{20} \)
- **B.** \( 1000^{20} \)
- **C.** \( P(1000, 20) = 1000 \cdot 9999 \cdots 9982 \cdot 9981 \)
- **D.** \( \binom{20 + 1000 - 1}{20} \)
- **E.** \( S(1000, 20) \)
M24a. Playlists: Given a hard drive with 1000 songs, how many ways can you make a playlist of 20 songs (repeats allowed)?

A. \( \binom{1000}{20} \)  

B. \( 1000^{20} \)  

C. \( P(1000, 20) = 1000 \cdot 9999 \cdots 9982 \cdot 9981 \)  

D. \( \binom{20 + 1000 - 1}{20} \)  

E. \( S(1000, 20) \)
M24b. Playlists: Given a hard drive with 1000 songs, how many ways can you make a playlist of 20 songs with no repeats?

A. \( \binom{1000}{20} \)  \quad B. 1000^{20}  \quad C. P(1000, 20) = 1000 \cdot 9999 \cdots 9982 \cdot 9981

D. \( \binom{20 + 1000 - 1}{20} \)  \quad E. S(1000, 20)
M24b. Playlists: Given a hard drive with 1000 songs, how many ways can you make a playlist of 20 songs with no repeats?

A. \( \binom{1000}{20} \)  
B. \( 1000^{20} \)  
C. \( P(1000, 20) = 1000 \cdot 9999 \cdots 9982 \cdot 9981 \)  
D. \( \binom{20 + 1000 - 1}{20} \)  
E. \( S(1000, 20) \)
Essay Question

You can prepare your answers ahead of time, like now.

- Describe something unique about yourself. What would set yourself apart from anyone else?
- If you had free time (which you don’t since you’re studying for midterms) what would you be doing for fun / culture?
- How do you plan on expanding your culture knowledge / experience (if at all)?
- What do you want to do after college?

Poll: Would you rather have:

A. More challenging quiz questions, but can use any resource, notes, book, internet, including ssh-ing into a supercomputer

B. Short simple quizzes, no calculators, no super computers,

Rubalcaba (rrrubalcaba@eng.ucsd.edu)
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C. Quizzes everyday, I want to take one now!!!
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D. No more quizzes!