http://vlsicad.ucsd.edu/courses/cse21-s14/
http://webwork.cse.ucsd.edu/webwork2/CSE21_Spring2014/
Homework 4 available
https://piazza.com/ucsd/spring2014/cse21/home
Topics for the week

- Functions
- Permutations (as functions & as actions)
  - cycle notation
  - composing permutations
  - The Enigma revisited
- Sterling Numbers (of the second kind) and Partitions
- Probability Distributions
- Random Graphs
- Buffon’s Needle
- Monty Hall Problem
- Computing large binomial coefficients
- Study sessions, weekend office hours.
Study Sessions with Rob, Tutors, & TAs - all ages welcome

- Week 4: Friday (4/25) 2-4pm at The Loft (HW4, general questions)
- TBD weekend study session
- Week 5: Wed. (4/30) 2-4pm at The Loft (Midterm Review)
Let \( f : A \to B \) be a function.

- If \( \forall b \in B \ \exists \geq 1 a \in A \ ( f(a) = b ) \) then \( f \) is a **surjection** or **onto** function.
- If \( \forall b \in B \ \exists \leq 1 a \in A \ ( f(a) = b ) \) then \( f \) is a **injection** or **one-to-one** function.
- If \( f \) is both an injection and a surjection it is called a **bijection**.
- If \( A = B \) and \( f \) is a bijection then it’s called a **permutation** of \( A \).
1. **Relations**: set of pairs

2. **Arrows**: domain set on left, codomain set on right
   - Translate each of the function properties to properties of these pictures.

3. **Two line notation**: matrix-like, with domain elements on top row and codomain elements on bottom.
   - Translate each of the function properties to properties of these pictures. In particular, what happens if the function is not surjective?

4. **Cycle notation**: *use only if $f$ is a permutation*
Flavor Flav in the house

\[ f(x) = x \mod 12 \]

Ex. \( 25 \mod 12 = 1 \)

= Remainder after dividing 13 by 12

\[
\begin{array}{c}
12 \sqrt{25} \\
-24
\end{array}
\]

\[
\begin{array}{c}
10 \\
9 \\
8 \\
7 \\
6 \\
5 \\
4 \\
3 \\
2 \\
1
\end{array}
\]
Functions

\[ f(x) = x \mod 4 \]

Example: \[ 21 \mod 4 = 1 \]

= Remainder after dividing 21 by 4

```
4 | 21
  | 20
  | 1
```
Functions

\[ f(x) = x \mod 4 \]

\[ f : \mathbb{Z} \rightarrow \{0, 1, 2, 3\} \]

\[ \text{DOMAIN} \quad \text{CODOMAIN} \]

\[ \ldots \]

\[ -1 \]

\[ 0 \quad \rightarrow \quad 0 \]

\[ 1 \quad \rightarrow \quad 1 \]

\[ 2 \quad \rightarrow \quad 2 \]

\[ 3 \quad \rightarrow \quad 3 \]

\[ 4 \]

\[ 5 \]

\[ 6 \]

\[ 7 \]

\[ \ldots \]
Permutations

\[
\begin{align*}
\begin{array}{c}
3 \\
2 \\
1 \\
4
\end{array}
\rightarrow
\begin{array}{c}
2 \\
1 \\
3 \\
4
\end{array}
\end{align*}
\]

\[
\begin{align*}
1 & \rightarrow 1 \\
2 & \rightarrow 2 \\
3 & \rightarrow 3 \\
4 & \rightarrow 4
\end{align*}
\]

\[
\begin{pmatrix}
1 & 2 & 3 & 4 \\
2 & 3 & 4 & 1
\end{pmatrix}
\]

\[
= (1, 2, 3, 4)
\]

\[
1 \rightarrow 2 \rightarrow 3 \rightarrow 4
\]
Permutations

\[ 3 \rightarrow 4 \rightarrow 2 \rightarrow 1 \]

\[ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} \]

\[ (1, 2) (3, 4) \]
Permutations

\[
\begin{array}{c}
\begin{array}{c}
3 \\
2 \\
4 \\
1 \\
\end{array}
\quad \rightarrow \\
\begin{array}{c}
1 \\
4 \\
2 \\
3 \\
\end{array}
\end{array}
\]

\[
\begin{array}{c}
1 \\
2 \\
3 \\
4 \\
\end{array}
\xrightarrow{function}

(1, 2, 3, 4)

(3, 4, 1, 2)

2 line notation
permutation
cycle notation

(1, 3)(2, 4)

1 \rightarrow 3 \rightarrow 1 \quad 2 \rightarrow 4 \rightarrow 2
Permutations

\[ R = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}, \quad F = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} \]

\[ R \circ F \quad (\text{Flip followed by rotation}) \]

\[ F \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} \]

\[ R \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix} \]

\[ R \circ F = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix} \text{ or } (1, 3) \]
Permutations

\[ R = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix} \]

\[ F = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} \]

\[ F \circ R \neq R \circ F = (1, 3) \]

\[ (1 \ 2 \ 3 \ 4) \]

\[ (1 \ 4 \ 3 \ 2) = (2, 4) \]
Is the function

\[ f : \{1, 2, 3\} \to \{1, 2, 3, 4\}, \quad f(x) = x + 1 \]

an injection, aka is it one-to-one?

A. Yes
B. No
C. ??
Is the function

\[ f : \{1, 2, 3\} \rightarrow \{1, 2, 3, 4\}, \quad f(x) = x + 1 \]

a surjection, aka is it onto?

A. Yes
B. No
C. ??
Properties of functions

Is the function

\[ f : \{1, 2, 3\} \rightarrow \{1, 2, 3, 4\}, \quad f(x) = x + 1 \]

a bijection?

A. Yes
B. No
C. ??
Properties of functions

Is the function

\[ f : \{1, 2, 3\} \rightarrow \{1, 2, 3, 4\}, \quad f(x) = x + 1 \]

A. Yes
B. No
C. ??

This function is one-to-one (injection), but not onto (note that nothing maps to 1 so it is not a surjection). Since it is not both a surjection and injection, this function is not a bijection.
Is the function

\[ f : \{1, 2, 3, 4\} \rightarrow \{1, 2, 3, 4\}, \]

\[ \begin{array}{c|c}
  x & f(x) \\
  \hline
  1 & 2 \\
  2 & 3 \\
  3 & 4 \\
  4 & 2 \\
\end{array} \]

a permutation?

A. Yes
B. No
C. ??

Note: it is not \textit{onto}.
What is the two-line (matrix) notation for the permutation

\[ g : \{1, 2, 3, 4\} \rightarrow \{1, 2, 3, 4\}, \]

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<tr>
<th>1</th>
<th>2</th>
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A. \[
\begin{pmatrix}
1 & 2 & 3 & 4 \\
1 & 2 & 3 & 4 \\
2 & 1 & 3 & 4 \\
2 & 1 & 3 & 4 \\
\end{pmatrix}
\]

B. \[
\begin{pmatrix}
1 & 2 & 3 & 4 \\
2 & 3 & 4 & 1 \\
1 & 4 & 3 & 2 \\
1 & 4 & 3 & 2 \\
\end{pmatrix}
\]

C. \[
\begin{pmatrix}
1 & 2 & 3 & 4 \\
2 & 3 & 4 & 1 \\
1 & 4 & 3 & 2 \\
1 & 4 & 3 & 2 \\
\end{pmatrix}
\]

D. \[
\begin{pmatrix}
1 & 2 & 3 & 4 \\
2 & 3 & 4 & 1 \\
1 & 4 & 3 & 2 \\
1 & 4 & 3 & 2 \\
\end{pmatrix}
\]

E. None of the above.
What is the (one-line) cycle notation for the permutation

\[
g : \{1, 2, 3, 4\} \rightarrow \{1, 2, 3, 4\},
\]

<table>
<thead>
<tr>
<th>x</th>
<th>g(x)</th>
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<tr>
<td>1</td>
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<tr>
<td>2</td>
<td>3</td>
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<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
</tbody>
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A. \((1, 2, 3, 4)\)

B. \((2, 3, 4, 1)\)

C. \((1, 2)(2, 3)(3, 4)(4, 1)\)

D. \((1, 2, 3)(4)\)

E. None of the above.
What is the composition of the following two permutations on \( \{1, 2, 3, 4, 5\} \)?

\[
f = (1, 2, 3)(4, 5) \quad g = (1)(2)(3, 4, 5)
\]

\[
f \circ g = ?
\]

A. \((1, 2, 3)(4, 5)\)
B. \((1)(2)(3, 4, 5)\)
C. \((1, 2, 4, 3)(5)\)
D. \((1, 2, 3, 5)(4)\)
E. None of the above.