UCSD CSE 21, Spring 2014

Mathematics for Algorithms and System Analysis

Week 5

Class URL: http://vlsicad.ucsd.edu/courses/cse21-s14/
Administrivia

• Attendance in this discussion is counted via clicker questions

• Homework Four is due TONIGHT!!!
  – Second chance!

• Midterm In-class on May 1 (ABK) and May 2 (RRR)
  – 30% of final grade
  – THIS WEEK!!!!
Administrivia

• This week:
  - Some homework 4 type questions
    • I’m going to talk about the key concepts on problems people are stuck on
  - Random Graphs
  - Derangements
  - Cumulative Density Functions
  - Some practice midterm problems
HW4 Question 2

• A warped coin has probability of 0.4 of landing Heads, probability of 0.3 of landing Tails, and probability 0.3 of landing on its Edge. It is flipped 5 times. What is the probability that more Heads occur than Tails?
  – How would you approach this problem?
Two teams A and B compete in a tournament consisting of at most 4 games. The probability that A wins any particular game is $2/3$ and the probability that B wins is $1/3$ (there can be no ties). If either team wins 2 consecutive games, that team is declared the winner. If at the end of 4 games there is no winner, the tournament is declared a draw.

- How would you approach this problem?
HW4 Questions 2 and 4

What do these two questions have in common?

A: They are both easier to solve if you split into cases
B: They both involve numbers less than one
C: What?
HW4 Questions 2 and 4

• What do these two questions have in common?
  A: They are both easier to solve if you split into cases
  B: They both involve numbers less than one
  C: What?
HW4 Question 10: Random Graphs

• Lots of questions on Piazza
• Also lots of responses on Piazza
• Anyone still have any questions about this?
Derangement

- Derangement is a permutation that requires ALL elements of the set to be changed.
Derangement

• Derangement is a permutation that requires ALL elements of the set to be changed.

• Suppose $S = \{1, 2, 3, 4, 5, 6\}$

• Is $(123)(456)$ a derangement of $S$?
  A: Yes   B: No   C: Yes   D: No
Derangement

• Is (1234) a derangement of S?
  A: Yes   B: No   C: Yes   D: No
Cumulative Density Function

• The CDF gives you the probability that the variable is less than a certain value
• Lets take a look at the bell curve (normal distribution)
• The CDFs are on the right.
Cumulative Density Function

• Basically: How much of the data is left of the top of the bell curve? Half of it is. So the value of the CDF at the mean is one half.
Cumulative Density Function

• Suppose X has the uniform distribution on [0,1], that is, X takes any value between 0 and 1 with equal probability. What is the CDF of X?

A) \( FX(x) = x \)  
B) \( FX(x) = x^2 \)  
C) \( FX(x) = 1 \)  
D) \( FX(x) = \frac{1}{2} \)  

E) \[
F_X(x) = \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{(y-\mu)^2}{2\sigma^2}} \, dy
\]
Midterm

DON'T PANIC,

ORGANISE!
Midterm review

• What is the coefficient of $x^5y^3$ in the expansion of $(3x + 4y)^8$
  - Discuss
Midterm review

• What is the coefficient of $x^5y^3$ $(3x + 4y)^8$
  
  \[ C(8,5)(3x)^5(4y)^8-5 \]
  
  \[ = C(8,5)(3^5)(4^3)x^5y^3 \]
  
  \[ = \text{a large number} \]
  
  \[ = 870912 \]
Midterm review

• Consider this distribution of probabilities for grabbing a random number of cookies from a jar:
  
  - $P(\text{get 0 cookies}) = \frac{1}{3}$
  - $P(\text{get 1 cookie}) = \frac{1}{6}$
  - $P(\text{get 2 cookies}) = \frac{1}{6}$
  - $P(\text{get 3 cookies}) = \frac{1}{3}$
Midterm review

• Consider this distribution of probabilities for grabbing a random number of cookies from a jar:
  - $P($get 0 cookies$) = 1/3$
  - $P($get 1 cookie$) = 1/6$
  - $P($get 2 cookies$) = 1/6$
  - $P($get 3 cookies$) = 1/3$
  - What is the expected number of cookies that will be grabbed?

$E[#\text{ of cookies}]$ is the sum of possible outcomes times their probabilities

Discuss
Midterm review

Consider this distribution of probabilities for grabbing a random number of cookies from a jar:

- \( P(\text{get 0 cookies}) = \frac{1}{3} \)
- \( P(\text{get 1 cookie}) = \frac{1}{6} \)
- \( P(\text{get 2 cookies}) = \frac{1}{6} \)
- \( P(\text{get 3 cookies}) = \frac{1}{3} \)
- What is the expected number of cookies that will be grabbed?

\[
E[\# \text{ of cookies}] = (0)(\frac{1}{3}) + (1)(\frac{1}{6}) + (2)(\frac{1}{6}) + (3)(\frac{1}{3})
\]

\[
= 0 + \frac{1}{6} + \frac{1}{3} + 1 = 1.5
\]
Midterm review

• Suppose you draw 7 cards without replacement from a standard deck of 52.
  - Let $X$ = the number of face cards and aces drawn
  - Let $Y$ = the number of black cards drawn
  - What is the expectation of $X+Y$ (written $E[X+Y]$)
  - First, $X$ and $Y$ are independent, so $E[X+Y] = E[X] + E[Y]$
Midterm review

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\[
E[X] = P(Ace)(\# \text{ of draws}) + P(King)(\# \text{ of draws}) + P(Queen)(\# \text{ of draws}) + P(Jack)(\# \text{ of draws})
= (4)(1/13)(7) = 28/13
\]

\[
E[Y] = P(\text{black card})(\# \text{ of draws}) = (1/2)(7) = 7/2 = 91/26
\]

\[
E[X] + E[Y] = 56/26 + 91/26 = 147/26
\]