UCSD CSE 21, Spring 2014

Mathematics for Algorithms and System Analysis

Week 4?

Class URL: http://vlsicad.ucsd.edu/courses/cse21-s14/
Administrivia

• Attendance in this discussion is counted via clicker questions
• Homework Four is due 4/27/2014
• Midterm In-class on May 1 (ABK) and May 2 (RRR)
  – 30% of final grade
• This week:
  – Functions (one-to-one, onto, bijective)
  – Counting Functions
  – Mop-up from CL
Mop-Up from CL

• Topics that were not carefully covered from the text
  – Stirling’s numbers of the second kind
  – Birthday paradox
Stirling’s numbers of the second kind

• Briefly: A Stirling number of the second kind (or Stirling partition number) is the number of ways to partition a set of $n$ objects into $k$ non-empty subsets and is denoted by $S(n,k)$.
Stirling’s numbers of the second kind

• Stirling: Set of size 3,
  \[ S(3,1) = 1 \]  
  \[ S(3,2) = 3 \]  
  \[ S(3,3) = 1 \]  
• See how it is different than choose?
Birthday Paradox

• See Prof. Khang’s upcoming lecture slides.
• Be aware of this problem
Functions

• Let $f$ be a function from the integers to the integers
  - Let $f(i) = |i|$ for every integer $i$
  - i.e, $f(-3) = 3$, $f(2) = 2$

• Is $f$ bijective?
  A: yes  B: no  C: yes  D: no  E: maybe
  (we’re not moving on until nobody answered E)
Functions

• Let \( f \) be a function from the integers to the integers
  - Let \( f(i) = |i| \) for every integer \( i \)
  - ie, \( f(-3) = 3, f(2) = 2 \)

• Is \( f \) onto?
  A: yes  B: no  C: yes  D: no  E: maybe
  (we’re not moving on until nobody answered E)
Functions

• Let f be a function from the integers to the integers
  - Let $f(i) = |i|$ for every integer $i$
  - ie, $f(-3) = 3$, $f(2) = 2$

• Is $f$ injective?
  A: yes  B: no  C: yes  D: no  E: maybe
  (we’re not moving on until nobody answered E)
Functions

• Let $f$ be a function from the integers to the integers
  - Let $f(i) = |i|$ for every integer $i$
  - ie, $f(-3) = 3$, $f(2) = 2$

• Is $f$ onto its range?
  A: yes  B: no  C: yes  D: no  E: maybe
  (we’re not moving on until nobody answered E)
Functions

• New function!!

• Let $f$ be a function from the integers to the **EVEN** integers
  - Let $f(i) = 2i$ for every integer $i$
  - i.e., $f(-3) = -6$, $f(2) = 4$

• Is $f$ bijective?
  A: yes  B: no  C: yes  D: no  E: maybe

(we’re not moving on until nobody answered E)
Functions

• Let f be a function from the integers to the **EVEN** integers
  - Let f(i) = 2i for every integer i
  - ie, f(-3) = -6, f(2) = 4

• Is f onto?
  A: yes  B: no  C: yes  D: no  E: maybe
  (we're not moving on until nobody answered E)
Functions

• Let $f$ be a function from the integers to the $EVEN$ integers
  - Let $f(i) = 2i$ for every integer $i$
  - ie, $f(-3) = -6$, $f(2) = 4$

• Is $f$ one-to-one?
  A: yes  B: no  C: yes  D: no  E: maybe
  (we’re not moving on until nobody answered E)
Functions

• Pictures Help:

A function
A one-to-one function (Not onto)
An onto function (Not one-to-one)
A bijection
Functions

• Facts to live by:
  – If f: A→ B is onto and g: B→ C is onto, then for a in A, then g\*f :A→ C is onto as well
  – If f is strictly increasing, ie, if x > y means that f(x)>f(y) (strictly!!) then f is one-to-one
Permutations

• Simplify this cycle notation permutation on the digits:
  \[(1 \ 2 \ 3)(2 \ 3 \ 4)(3 \ 4 \ 5)(4 \ 5 \ 6)\]
  = ?