Midterm Friday
One 8.5 x 11 sheet (both sides) of *handwritten* notes, no electronics

Scattered, Smothered, and Covered
### Office Hours / Study Sessions this week

- **Wed. (4/30) 2-4:00pm** at *The Loft* (Midterm Q&A w/ TAs, tutors)
- **Wed. (4/30) 6-7:00pm** at *EBU3B 4122*
- **Thurs. (5/1) 3-4pm** *EBU3B Basement*
- **Thurs. (5/1) 7-9pm** *CENTER 119* (Midterm Q&A)
- **Thurs. (5/1) 9-11pm** *Porters Pub - (tentative)*
- **Friday (5/2) 12-12:40** *EBU3B 4122*
Sample Midterm Questions

1. List all derangements of the four element set \{1, 2, 3, 4\} in cycle notation.
2. What is the probability that a random permutation on four letters is a derangement?
3. Find \((\{(2, 3, 4, 5) \circ (1, 3, 5) \circ (1, 2)(3, 5)\})^{-1}\)
4. Given an alphabet of just 6 letters, \{A, B, C, D, E, F\} how many plugboard wires would give the most possible arrangements?
5. How many ways are there to connect two plugboard wires with a restricted alphabet of five letters? How does this relate to Sterling Numbers of the second kind (explain the connection or lack of connection)?
6. How many ways are there to solve \(w+x+y+z=10\), if each of \(w, x, y, z\) are positive integers?
7. What is the probability of getting a full house dealt from standard deck of 52 cards?
8. How many partitions of a set of five labeled elements \{A, B, C, D, E\} have two unlabeled subsets?
9. In a class with 22 students, what is the probability that at least two students share a birthday?
10. In a class with 22 students, what is the probability that no students share a birthday?
11. How many injective functions are there from \(D = \{1, 2, 3\}\) to \(C = \{a, b, g, y, z\}\)?
12. How many surjective functions are there from \(D = \{1, 2, 3, 4\}\) to \(C = \{\text{red, blue}\}\)?
13. Encrypt this message with the Caesar cipher: MIDTERM
14. What is the probability that a random graph \(G(15, 0.2)\) has 15 edges? What is the expected number of edges?
15. List all partitions of five labeled elements that have three unlabeled subsets of size 3,1, and 1. Is this \(S(5, 3)\)?
16. Calculate the number of ways to partition a set of five labeled objects into three labeled subsets of size 3,1,1 in two different ways, one with multinomial coefficients and one way with binomial coefficients
17. Given \( P(A) = \frac{3}{10} \), \( P(A \cup B) = \frac{9}{10} \), \( P(A \cap B) = \frac{7}{10} \). Find \( P(B) \).

18. There are 10 light bulbs, 2 of which are defective. If you randomly select 3 bulbs, what is the probability that you will get two working light bulbs and one defective bulb?

19. A local UCSD organization held a raffle to raise money. They sold $1 raffle tickets for an iPad valued at $300. If they sold 1000 tickets, what is the expected value of one raffle ticket?

20. Let \( D \) be a \textit{directed} graph on \( N \) vertices, how many directed edges are possible?

21. \textit{Deep Thoughts:} Is the composition of two permutations on \( n \) labeled objects \textit{always} another permutation on the same \( n \) labeled objects? Can two different letters (such as \( E \) and \( Z \)) be mapped to the same letter (such as \( M \)) after composing two permutations on the set \{\( A, B, \ldots, Z \)\}?

22. Count number of permutations on six letters that fix one letter, fix two letters.

23. Hash browns at Waffle House always come ”scattered (on the grill)” but you can also ask for them to be: smothered, covered, chunked, diced, peppered, capped, topped, or country. You can ask for at most one of any of these extras, for example the typical Waffle House order would ask for hash browns ”Scattered, Smothered, and Covered.” How many ways can you order hash browns at Waffle House?

24. Playlists: Given a hard drive with 1000 songs, how many ways can you make a playlist of 20 songs (repeats allowed)? How many ways can you make a playlist of 20 songs with no repeats?

25. Essay Question (see last slide)
M1. List all derangements of the four element set \( \{1, 2, 3, 4\} \) in cycle notation.

A derangement is a permutation on the set \( S \) where no element is mapped to itself, that is \( f(x) \neq x \) for all \( x \in S \).

There are three choices where to send 1 namely, 2, 3, or 4. For each choice of where 1 is sent, there are three choices where to send \( X \) (we can’t send \( X \) to itself). The third and fourth maps are forced. The following ordering illustrates this:

1 → 2, 2 →?
1 → 3, 3 →?
1 → 4, 4 →?

\( (1, 2)(3, 4) \quad (1, 2, 3, 4) \quad (1, 2, 4, 3) \)
\( (1, 3)(2, 4) \quad (1, 3, 2, 4) \quad (1, 3, 4, 2) \)
\( (1, 4)(2, 3) \quad (1, 4, 2, 3) \quad (1, 4, 3, 2) \)

You can also compute \( !4 \) using the recursive formula (where \( !0 = 1 \) and \( !1 = 0 \)).

\[
!n = (n - 1)(!(n - 1) + !(n - 2)) \\
!2 = (2 - 1)(!(2 - 1) + !(2 - 2)) = 1 \\
!3 = (3 - 1)(!(3 - 1) + !(3 - 2)) = 2 \\
!4 = (4 - 1)(!(4 - 1) + !(4 - 2)) = 3 \cdot 3 = 9
\]
M2. What is the probability that a random permutation on four letters is a derangement?

There are $4! = 24$ permutations on 4 letters, say \{E, R, Q, W\}. From Question 1, we saw there were 9 derangements.

\[
\begin{align*}
  (E, R)(Q, W) & \quad (E, R, Q, W) & \quad (E, R, W, Q) \\
  (E, Q)(R, W) & \quad (E, Q, R, W) & \quad (E, Q, W, R) \\
  (E, W)(R, Q) & \quad (E, W, R, Q) & \quad (E, W, Q, R)
\end{align*}
\]

The probability of randomly selecting a permutation on four letters is $\frac{9}{24}$. 
M3. Find \(((2, 3, 4, 5) \circ (1, 3, 5) \circ (1, 2)(3, 5))^{-1}\)

Let \(f = (2, 3, 4, 5), g = (1, 3, 5), h = (1, 2)(3, 5)\). First we compute
\(f \circ g \circ h = (2, 3, 4, 5) \circ (1, 3, 5) \circ (1, 2)(3, 5)\)

- \(3 \overset{f}{\leftarrow} 2 \overset{g}{\leftarrow} 2 \overset{h}{\leftarrow} 1\)
- \(4 \overset{f}{\leftarrow} 3 \overset{g}{\leftarrow} 1 \overset{h}{\leftarrow} 2\)
- \(1 \overset{f}{\leftarrow} 1 \overset{g}{\leftarrow} 5 \overset{h}{\leftarrow} 3\)
- \(5 \overset{f}{\leftarrow} 4 \overset{g}{\leftarrow} 4 \overset{h}{\leftarrow} 4\)
- \(2 \overset{f}{\leftarrow} 5 \overset{g}{\leftarrow} 3 \overset{h}{\leftarrow} 5\)
Let \( f = (2, 3, 4, 5), g = (1, 3, 5), h = (1, 2)(3, 5) \). First we compute 
\[ f \circ g \circ h = (2, 3, 4, 5) \circ (1, 3, 5) \circ (1, 2)(3, 5) \]
So we have:

- \( 3 \leftarrow f \circ g \circ h \) 1
- \( 4 \leftarrow f \circ g \circ h \) 2
- \( 1 \leftarrow f \circ g \circ h \) 3
- \( 5 \leftarrow f \circ g \circ h \) 4
- \( 2 \leftarrow f \circ g \circ h \) 5
M3. Find \(((2, 3, 4, 5) \circ (1, 3, 5) \circ (1, 2)(3, 5))^{-1}\)

To find the inverse, reverse the arrows:

- \(3 \rightarrow (f \circ g \circ h)^{-1} 1\)
- \(4 \rightarrow (f \circ g \circ h)^{-1} 2\)
- \(1 \rightarrow (f \circ g \circ h)^{-1} 3\)
- \(5 \rightarrow (f \circ g \circ h)^{-1} 4\)
- \(2 \rightarrow (f \circ g \circ h)^{-1} 5\)

To write this in cycle notation observe: \(1 \rightarrow 3 \rightarrow 1 \quad 2 \rightarrow 5 \rightarrow 4 \rightarrow 2\)

\(((2, 3, 4, 5) \circ (1, 3, 5) \circ (1, 2)(3, 5))^{-1} = (1, 3)(2, 5, 4)\)
M4. Given an alphabet of just 6 letters, \{A, B, C, D, E, F\}. How many plugboard wires would give the most possible arrangements?

There are 0, 1, 2, or 3 plugboard wires possible.

- 0 plugboard wires: There is one way to use no plugboard wires, just don’t use any. This can be described by the permutation 

- 1 plugboard wire pair: \(C(6, 2) = \binom{6}{2} = \frac{6\cdot5\cdot4}{2\cdot1} = 15\) Let’s list them all in cycle notation lexicographically (dictionary order):

\[
\{(AB), (AC), (AD), (AE), (AF), \\
(BC), (BD), (BE), (BF), (CD), \\
(CE), (CF), (DE), (DF), (EF)\}
\]
M4. Given an alphabet of just 6 letters, \{A, B, C, D, E, F\}. How many plugboard wires would give the most possible arrangements?

- 2 plugboard wires: \(C(6, 2) = \binom{6}{2}\) for the first pair, times \(C(4, 2) = \binom{4}{2}\) ways to pick the next pair. We are over counted by a factor 2!, since we double counted the indistinguishable wires.

\[
\frac{\binom{6}{2}\binom{4}{2}}{2!} = \frac{90}{2} = 45
\]

M4. Given an alphabet of just 6 letters, \( \{A, B, C, D, E, F\} \). How many plugboard wires would give the most possible arrangements?

There are 0, 1, 2, or 3 plugboard wires possible.

- 3 plugboard wires:

\[
\binom{6}{2} \binom{4}{2} \binom{2}{2} \div 3! = \frac{90}{3!} = 15
\]

\[
\{(AB)(CD)(EF), (AB)(CE)(DF), (AB)(CF)(DE),
(AC)(BD)(EF), (AC)(BE)(DF), (AC)(BF)(ED),
(AD)(BC)(EF), (AD)(BE)(CF), (AD)(BF)(CE),
(AF)(BC)(DE), (AF)(BD)(CE), (AF)(BE)(CD)\}
\]

Two plugboard wires gives the largest number of configurations (45).
M5. How many ways are there to connect two plugboard wires with a restricted alphabet of five letters? How does this relate to Sterling Numbers of the second kind (explain the connection or lack of connection)?

Note if our restricted alphabet is the first five letters \{A, B, C, D, E\},

\[(AB)(CD), \ (AB)(CE), \ (AB)(DE), \ (AC)(BD), \ (AC)(BE), \]
\[(AC)(DE), \ (AD)(BC), \ (AD)(BE), \ (AD)(CE), \ (AE)(BC), \]
\[(AE)(BD), \ (AE)(CD), \ (BC)(DE), \ (BD)(CE), \ (BE)(CD)\]

\[
\left(\binom{5}{2}\right)\left(\binom{3}{2}\right) \div 2! = 30/2! = 15
\]
M5. How many ways are there to connect two plugboard wires with a restricted alphabet of five letters? How does this relate to Sterling Numbers of the second kind (explain the connection or lack of connection)?

There are \( \frac{5 \cdot 3}{2!} \) = 30/2! = 15 ways. This is only part of calculation of \( S(5, 3) \) where we write 5 = 2 + 2 + 1 since plugboard wires are unlabeled subsets of size two and letters without wires are unlabeled subsets of size one.
M6. How many ways are there to solve \( w + x + y + z = 10 \), if each of \( w, x, y, z \) are positive integers?

This is a stars and bars problem, with 10 stars and three bars, but with each variable starting with 4 stars.

\[
\begin{array}{cccc}
  w & x & y & z \\
  \star & \star & \star & \star \\
\end{array}
\]

Now we have 6 stars left: here is one possible solution

\[
\begin{array}{cccc}
  w & x & y & z \\
  \star \star \star & \star & \star & \star \\
\end{array}
\]

(corresponding to

\[
\begin{array}{cccc}
  w & x & y & z \\
  \star \star \star & \star & \star & \star \star \star \\
\end{array}
\]

So we have six stars and three bars, this is \( \binom{6 + 3}{3} \) or

\[
\binom{\text{stars + bars}}{\text{bars}} = \binom{6 + 3}{3} = \binom{9}{3} = \frac{9 \cdot 8 \cdot 7}{3 \cdot 2 \cdot 1} = 84
\]
What is the probability of getting a full house dealt from standard deck of 52 cards?

First we choose the rank (13 ways) for the three of a kind, then assign suits to those three cards \( C(4, 3) = \binom{4}{3} = 4 \) ways, then pick the rank for the pair (12 ways), and assign suits \( C(4, 2) = \binom{4}{2} = 6 \) ways to those cards. Then we divide by the total number of ways to draw five cards from 52 without replacement.

\[
P(\text{full house}) = \frac{13 \cdot 12 \cdot 4 \cdot 4 \cdot 4}{52 \cdot 5} \approx 0.001441
\]
M8. How many partitions of a set of five labeled elements \( \{A, B, C, D, E\} \) have two unlabeled subsets?

\[
S(5,2) = 10 + 5 = 15
\]

\[
5 = 3 + 2 \\
5 = 4 + 1
\]
M9. In a class with 22 students, what is the probability that at least two students share a birthday?

Let’s assume 365 possible birthdays (sorry to anyone with Feb 29th as their birthday) and every birthday is equally likely.

Let \( D(N) \) be the probability that in a group with \( N \) people no two people share the same birthday. The event of at least two of the \( N \) persons having the same birthday is complementary to all \( N \) birthdays being different. Thus, \( B(N) = 1 - D(N) \) is the probability that in a group with \( N \) people at least two people share the same birthday.

- \( D(1) = \frac{365}{365} = 1 \)
- \( D(2) = \frac{365}{365} \cdot \frac{364}{365} \) (There are 364 choices for birthdays other than the first person).
- \( D(3) = \frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \)
- \( D(N) = \frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \cdot \ldots \cdot \frac{365-N+1}{365} = \frac{P(365,N)}{365^N} \)

\[
B(22) = 1 - \frac{P(365, 22)}{365^{22}} \approx 0.4757
\]
M10. In a class with 22 students, what is the probability that no students share a birthday?

We answered this whilst answering the previous question.

\[
\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \cdot \ldots \cdot \frac{365 - N + 1}{365} = \frac{P(365, 22)}{365^{22}} \approx 0.5243
\]
M11. How many injective functions are there from $D = \{1, 2, 3\}$ to $C = \{a, b, g, y, z\}$?

There are five choices where to send 1, once a choice is made for $1 \to ?$, four choices where to send 2, and three choices of where to send 3. This is $P(5, 3) = 5 \cdot 4 \cdot 3 = 60$. 
M12. How many surjective functions are there from $D = \{1, 2, 3, 4\}$ to $C = \{\text{red, blue}\}$?

One solution is using Sterling numbers of the second kind. $S(4, 2) \cdot 2! = 7 \cdot 2 = 14$. We are partitioning the domain into two sets, one partition will go to Blue and one partition will go to Red. Then we multiply by $2!$ to consider all rearrangements of Red and Blue.

For this example it’s easiest to count the number of functions that are not onto. For a function with codomain of size two to not be onto, all of the elements must be sent to red or blue (the image should have only one element). There are only two such functions, either everything is sent to red or everything is sent to blue. There are $2^4$ possible functions, so $2^4 - 2 = 14$ are surjective.
M13. "Encrypt this message with the Caesar cipher: MIDTERM"

Shift each letter by three: \( M \rightarrow P, I \rightarrow L, D \rightarrow G, T \rightarrow W, E \rightarrow H, R \rightarrow U, M \rightarrow P \)

\[
\begin{array}{cccccccccccccccccccc}
\end{array}
\]

PLGWHUP
M14. What is the probability that a random graph $G(15, 0.2)$ has 15 edges? What is the expected number of edges?

$$\binom{105}{15} (0.2)^{15} (1 - 0.2)^{105 - 15} = \binom{105}{15} (0.2)^{15} (0.8)^{90}$$

possible edges.

Edges are formed by independent coin tosses which are binomial distributed (with $n=105$ and $p=0.2$), so the expected value (where $X$ is the random variable \# of edges) is

$$E(X) = 105 \cdot 0.2 = 21$$
M14a. What is the probability that a random graph $G(15, 0.2)$ has 19 edges? What is the expected number of edges?

$$\binom{105}{19} (0.2)^{19} (1 - 0.2)^{105 - 19} = \binom{105}{19} (0.2)^{19} (0.8)^{86}$$ possible edges.

Edges are formed by independent coin tosses which are binomial distributed (with $n=105$ and $p=0.2$), so the expected value (where $X$ is the random variable # of edges) is $E(X) = 105 \cdot 0.2 = 21$. 
M15. List all partitions of five labeled elements that have three unlabeled subsets of size 3, 1, and 1. Is this $S(5, 3)$?

Let's label the elements of the set $\{A, B, C, D, E\}$:

- $\{A, B, C\}, \{D\}, \{E\}$
- $\{B, C, D\}, \{A\}, \{E\}$
- $\{C, D, E\}, \{A\}, \{B\}$
- $\{D, E, A\}, \{B\}, \{C\}$
- $\{E, A, B\}, \{C\}, \{D\}$
- $\{A, C, D\}, \{B\}, \{E\}$
- $\{B, D, E\}, \{A\}, \{C\}$
- $\{C, E, A\}, \{B\}, \{D\}$
- $\{D, A, B\}, \{C\}, \{E\}$
- $\{E, B, C\}, \{A\}, \{D\}$

This is not $S(5, 3)$, however, this is only part of the calculation of $S(5, 3)$, note there are other ways of partitioning a set of five labeled objects into three unlabeled subsets, such as $5 = 2 + 2 + 1$. 
M16. Calculate the number of ways to partition a set of five labeled objects into three labeled subsets of size 3,1,1 in two different ways, one with multinomial coefficients and one way with binomial coefficients.

\[
\binom{5}{3, 1, 1} = \frac{5!}{3!1!1!} = 20
\]

Note that since these are labeled subsets, once we choose the three elements to be in one partition of size 3, there are two choices for the remaining two elements to be assigned to the two labeled subsets.

\[
\begin{align*}
\binom{5}{3} \cdot 2 &= 10 \cdot 2 = 20 \\
\end{align*}
\]
M17. Given $P(A) = \frac{3}{10}$, $P(A \cap B)^c = \frac{9}{10}$, $P(A \cup B) = \frac{7}{10}$. Find $P(B)$.
(Note diagram is not drawn to scale).

\[
P(A \cup B) = P(A) + P(B) - P(A \cap B)
\]

A. 0.3    B. 0.4    C. 0.5    D. 0.6    E. 0.7
M17. Given \( P(A) = \frac{3}{10} \), \( P(A \cap B)^c = \frac{9}{10} \), \( P(A \cup B) = \frac{7}{10} \). Find \( P(B) \).

(Note diagram is not drawn to scale).

\[
P(A \cup B) = P(A) + P(B) - P(A \cap B)
\]

\[
0.7 = 0.3 + P(B) - (1 - 0.9)
\]

\[
0.7 = 0.3 + P(B) - 0.1
\]

\[
0.7 - 0.3 + 0.1 = P(B)
\]

A. 0.3  B. 0.4  C. 0.5  D. 0.6  E. 0.7
M18. There are 10 light bulbs, 2 of which are defective. If you randomly select 3 bulbs, what is the probability that you will get two working light bulbs and one defective bulb?
**M18.** There are 10 light bulbs, 2 of which are defective. If you randomly select 3 bulbs, what is the probability that you will get two working light bulbs and one defective bulb?
There are 8 good bulbs from which you will select 2, and 2 bad bulbs of which you will select 1.
M18. There are 10 light bulbs, 2 of which are defective. If you randomly select 3 bulbs, what is the probability that you will get two working light bulbs and one defective bulb?
There are 8 good bulbs from which you will select 2, and 2 bad bulbs of which you will select 1.

\[ \frac{\binom{2}{1} \cdot \binom{8}{2}}{\binom{10}{3}} = \frac{28 \cdot 2}{120} \approx 0.4667 \]
Midterm Practice Question 19

**M19.** A local UCSD organization held a raffle to raise money. They sold $1 raffle tickets for an iPad valued at $300. If they sold 1000 tickets, what is the expected value of one raffle ticket?

Let $X$ be the amount won or lost. If you don’t win the iPad you lose one dollar, or $X_1 = -1$. If you win the iPad you have won the equivalent of $300$ minus the one dollar you spent on the ticket.
M19. A local UCSD organization held a raffle to raise money. They sold $1 raffle tickets for an iPad valued at $300. If they sold 1000 tickets, what is the expected value of one raffle ticket?

Let $X$ be the amount won or lost. If you don’t win the iPad you lose one dollar, or $X_1 = -1$. If you win the iPad you have won the equivalent of $300 minus the one dollar you spent on the ticket.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$f(X)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$300 - 1$</td>
<td>$\frac{1}{1000} = 0.001$</td>
</tr>
<tr>
<td>$-1$</td>
<td>$\frac{999}{1000} = 0.999$</td>
</tr>
</tbody>
</table>
**M19.** A local UCSD organization held a raffle to raise money. They sold $1 raffle tickets for an iPad valued at $300. If they sold 1000 tickets, what is the expected value of one raffle ticket?

Let $X$ be the amount won or lost. If you don’t win the iPad you lose one dollar, or $X_1 = -1$. If you win the iPad you have won the equivalent of $300 minus the one dollar you spent on the ticket.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$f(X)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>300-1</td>
<td>$\frac{1}{1000} = 0.001$</td>
</tr>
<tr>
<td>-1</td>
<td>$\frac{999}{1000} = 0.999$</td>
</tr>
</tbody>
</table>

$$E(X) = \sum X_i \cdot P(X_i)$$

where the sum is taken over all possible values of $X_i$.

$$E(X) = \$299 \cdot (0.001) + (-\$1)(0.999) = -\$0.70$$
M20. Let D be a directed graph on \( N \) vertices, how many directed edges are possible?

Each pair of vertices has two directed edges possible, one from \( v_a \) to \( v_b \) and one from \( v_b \) to \( v_a \).

A. \( C(N,2) \)  
B. \( 2 \cdot C(N,2) \)  
C. \( 2 \cdot P(N,2) \)  
D. \( P(N,2) \)
M20. Let $D$ be a directed graph on $N$ vertices, how many directed edges are possible?

Each pair of vertices has two directed edges possible, one from $v_a$ to $v_b$ and one from $v_b$ to $v_a$. So twice as many directed edges as an undirected graph (undirected graphs can have at most $\binom{N}{2}$ undirected edges).

Or, we are counting how many ordered pairs there are from $N$ objects.

A. $\binom{N}{2}$  
B. $2 \cdot \binom{N}{2}$  
C. $2 \cdot P(N, 2)$  
D. $P(N, 2)$
M21. Deep Thoughts: Is the composition of two permutations on $n$ labeled objects always another permutation on the same $n$ labeled objects? Can two different letters (such as $E$ and $Z$) be mapped to the same letter (such as $M$) after composing two permutations on the set $\{A, B, ..., Z\}$?
M22. Count number of permutations on six letters \( \{ Q, W, E, R, T, Y \} \) that fix the letter \( Q \).

\[
\begin{pmatrix}
Q & W & E & R & T & Y \\
Q & * & * & * & * & *
\end{pmatrix}
\]

A. \( 6! \)    B. \( 6! - 1! \)    C. \( 5! \)    D. \( \frac{6!}{1!} \)    E. ??
M22. Count number of permutations on six letters \{Q, W, E, R, T, Y\} that fix the letter Q.

\[
\begin{pmatrix}
Q & W & E & R & T & Y \\
Q & * & * & * & * & *
\end{pmatrix}
\]

A. 6!  
B. 6! − 1!  
C. 5!  
D. \( \frac{6!}{1!} \)  
E. ??

Note this is the same as counting all permutations on five letters \{W, E, R, T, Y\}
M22. Count number of permutations on six letters \{Q, W, E, R, T, Y\} that fix the letters T and Y.

\[
\begin{pmatrix}
Q & W & E & R & T & Y \\
* & * & * & * & T & Y
\end{pmatrix}
\]

A. 6!  
B. 6! – 2!  
C. 4!  
D. \(\frac{6!}{2!}\)  
E. ??
**M22.** Count number of permutations on six letters \( \{Q, W, E, R, T, Y\} \) that fix the letters \( T \) and \( Y \).

\[
\begin{pmatrix}
  Q & W & E & R & T & Y \\
  * & * & * & * & T & Y
\end{pmatrix}
\]

A. \( 6! \)  
B. \( 6! - 2! \)  
C. \( 4! \)  
D. \( \frac{6!}{2!} \)  
E. ??

Note this is the same as counting all permutations on *four* letters \( \{Q, W, E, R\} \)
M23a. Hash browns at waffle house always come ”scattered” (spread on the grill), but you can also ask for them to be:

- ”smothered” (with onions)
- ”covered” (with cheese)
- ”chunked” (with diced ham)
- ”diced” (with diced tomatoes)
- ”peppered” (with jalapeo peppers)
- ”capped” (with mushrooms)
- ”topped” (with Bert’s chili)
- ”country” (with sausage gravy)

How many ways are there to ask for a single plate of hash browns at Waffle House (at most one of each extra, no repeats)?

A. $8!$  
B. $2^8$  
C. $\binom{2 + 8 - 1}{8 - 1}$  
D. $S(8, 2)$  
E. $\binom{8}{2}$
M23a. Hash browns at waffle house always come ”scattered” (spread on the grill), but you can also ask for them to be:

- "smothered” (with onions)
- "covered” (with cheese)
- "chunked” (with diced ham)
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- "capped” (with mushrooms)
- "topped” (with Bert's chili)
- "country” (with sausage gravy)

How many ways are there to ask for a single plate of hash browns at Waffle House (at most one of each extra, no repeats)? Each extra has 2 possibilities: either add the extra or don’t, so there are __ possible ways.

A. 8!  B. $2^8$  C. $\binom{2 + 8 - 1}{8 - 1}$  D. $S(8, 2)$  E. $\binom{8}{2}$
Midterm Practice Question 23

M23b. Hash browns at waffle house always come “scattered” (spread on the grill), but you can also ask for them to be:

- “smothered” (with onions)
- “covered” (with cheese)
- “chunked” (with diced ham)
- “diced” (with diced tomatoes)
- “peppered” (with jalapeño peppers)
- “capped” (with mushrooms)
- “topped” (with Bert’s chili)
- “country” (with sausage gravy)

How many ways are there to ask for a single plate of hash browns at Waffle House with exactly two extras (no repeats)?

A. $8!$ B. $2^8$ C. $\binom{2 + 8 - 1}{8 - 1}$ D. $S(8, 2)$ E. $\binom{8}{2}$
M23b. Hash browns at waffle house always come ”scattered” (spread on the grill), but you can also ask for them to be:

- ”smothered” (with onions)
- ”covered” (with cheese)
- ”chunked” (with diced ham)
- ”diced” (with diced tomatoes)
- ”peppered” (with jalapeño peppers)
- ”capped” (with mushrooms)
- ”topped” (with Bert’s chili)
- ”country” (with sausage gravy)

How many ways are there to ask for a single plate of hash browns at Waffle House with exactly two extras (no repeats)? *Here you are choosing 2 things from 8, without regard to order.*

A. $8!$  B. $2^8$  C. $\binom{2 + 8 - 1}{8 - 1}$  D. $S(8, 2)$  E. $\binom{8}{2}$
M23c. Hash browns at waffle house always come ”scattered” (spread on the grill), but you can also ask for them to be:

- ”smothered” (with onions)
- ”covered” (with cheese)
- ”chunked” (with diced ham)
- ”diced” (with diced tomatoes)
- ”peppered” (with jalapeo peppers)
- ”capped” (with mushrooms)
- ”topped” (with Bert’s chili)
- ”country” (with sausage gravy)

How many ways are there to ask for a single plate of hash browns at Waffle House with exactly two extras (repeats allowed: i.e. you can ask for covered twice)?

A. 8!  B. $2^8$  C. $\binom{2 + 8 - 1}{8 - 1}$  D. $S(8, 2)$  E. $\binom{8}{2}$
M23c. Hash browns at waffle house always come "scattered" (spread on the grill), but you can also ask for them to be:

- "smothered" (with onions)
- "covered" (with cheese)
- "chunked" (with diced ham)
- "diced" (with diced tomatoes)
- "peppered" (with jalapeño peppers)
- "capped" (with mushrooms)
- "topped" (with Bert's chili)
- "country" (with sausage gravy)

How many ways are there to ask for a single plate of hash browns at Waffle House with exactly two extras (repeats allowed: i.e. you can ask for covered twice)? *Stars and bars.* 7 bars, 2 stars. You can also do $8*7/2! +$ the 8 repeats, but this method becomes tedious for larger values of "2".

A. $8!$  
B. $2^8$  
C. $\binom{2 + 8 - 1}{8 - 1}$  
D. $S(8, 2)$  
E. $\binom{8}{2}$
M24a. Playlists: Given a hard drive with 1000 songs, how many ways can you make a playlist of 20 songs (repeats allowed)?

A. \( \binom{1000}{20} \)  
B. \( 1000^{20} \)  
C. \( P(1000, 20) = 1000 \cdot 9999 \cdots 9982 \cdot 9981 \)  
D. \( \binom{20 + 1000 - 1}{20} \)  
E. \( S(1000, 20) \)
M24a. Playlists: Given a hard drive with 1000 songs, how many ways can you make a playlist of 20 songs (repeats allowed)?

A. \( \binom{1000}{20} \)  
B. \( 1000^{20} \)  
C. \( P(1000, 20) = 1000 \cdot 9999 \cdot \ldots \cdot 9982 \cdot 9981 \)  
D. \( \binom{20 + 1000 - 1}{20} \)  
E. \( S(1000, 20) \)
M24b. Playlists: Given a hard drive with 1000 songs, how many ways can you make a playlist of 20 songs with no repeats?

A. \( \binom{1000}{20} \)  
B. \( 1000^{20} \)  
C. \( P(1000, 20) = 1000 \cdot 9999 \cdot \ldots \cdot 9982 \cdot 9981 \)  
D. \( \binom{20 + 1000 - 1}{20} \)  
E. \( S(1000, 20) \)
M24b. Playlists: Given a hard drive with 1000 songs, how many ways can you make a playlist of 20 songs with no repeats?

A. \( \binom{1000}{20} \)  
B. \( 1000^{20} \)  
C. \( P(1000, 20) = 1000 \cdot 9999 \cdot \ldots \cdot 9982 \cdot 9981 \)  
D. \( \binom{20 + 1000 - 1}{20} \)  
E. \( S(1000, 20) \)
Essay Question

You can prepare your answers ahead of time, like now.

- Describe something unique about yourself.
- How do you plan on expanding your culture knowledge / experience (if at all)?
- What do you want to do after college?