CSE 101 Practice Problems - big-O notation

Running-Time Analysis

For the pseudo-code below, give the asymptotic running time in \( \Theta \) notation:

1. for \( i = 1 \) to \( n \) do
   \( j = i \)
   while \( j < n \) do
      \( j = j + 3 \)

   Answer: The running time is approximately the following sum: \( \sum_{i=1}^{n} (n-i)/3 \) which is \( \Theta(n^2) \).

2. for \( i = 1 \) to \( n \) do
   for \( j = 2*i \) to \( n \) do
      \( s = s+1 \)

   Answer: The running time is approximately the following sum: \( \sum_{i=1}^{n/2} n - 2i + \sum_{i=n/2+1}^{n} 1 \) which is \( \Theta(n^2) \).

3. for \( i = 1 \) to \( n \) do
   \( j = i \)
   while \( j < n \) do
      \( j = 2*j \)

   Answer: The running time is approximately the following sum: \( \sum_{i=1}^{n} 1 + \log(n/i) \) which is \( \Theta(n) \).

4. for \( i = 1 \) to \( n \) do
   \( j = n \)
   while \( i*i < j \) do
      \( j = j - 1 \)

   Answer: The running time is approximately the following sum: \( \sum_{i=1}^{\sqrt{n}} n - i^2 + \sum_{i=\sqrt{n}+1}^{n} 1 \) which is \( \Theta(n^{3/2}) \).

5. for \( i = 1 \) to \( n \) do
   \( j = 2 \)
   while \( j < i \) do
      \( j = j*j \)

   Answer: The running time is approximately the following sum: \( \sum_{j=2}^{n} \log j = \log \prod_{j=2}^{n} j \log j \) The log of the product can be approximated with \( \Theta(n \log \log n) \).

(source: Berkeley CS 170, Fall 2009, HW 1)