

## Optimal Planning for Mesh-Based Power Distribution\*

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**Abstract**— Robust power distribution within available routing area resources is critical to chip performance and reliability. In this paper, we propose a novel and efficient method for optimizing worst-case static IR-drop in hierarchical, uniform power distribution networks. Our results can be used for *planning of hierarchical power distribution in early design stages*, so that for a fixed total routing area the worst-case IR-drop on the power mesh is minimal, or for a given IR-drop tolerance the power mesh achieves the IR-drop specification with minimal routing area. Our contributions are as follows. (1) We derive a closed-form approximation for the worst-case IR-drop on a single-level power mesh. The formula shows that for a given total routing area, the worst-case IR-drop increases logarithmically with the number of metal lines on the mesh. (2) Based on the previous analysis and empirical studies, we propose a model for the worst-case static IR-drop on a two-level power mesh, and obtain an accurate empirical expression. (3) Using this expression, we present a novel approach to optimize the two-level mesh topology. (4) We extend our study to three-level power meshes, and find that a third, middle-level mesh helps to reduce IR-drop by only a relatively small extent (about 5%, according to our experiments).

### I. INTRODUCTION

Higher device density and faster switching frequencies result in larger currents flowing through the power distribution network; IR-drop thus becomes a significant problem. Simply using more routing area for the power network may cause serious routability problems. Hence, robust power distribution within available area resources becomes vital to achieving performance and reliability in high-end VLSI designs.

In engineering practice, design of the power distribution network usually consists of a number of stages. Many important decisions – notably, the nominal wiring pitch and width for each interconnect layer – are locked in for the power distribution network very early in the design process. In early stages of design, the power network has not yet been synthesized and the location and logic content of the blocks are unknown. It is therefore impossible to obtain the pattern of current drawn by the sinks, and transient analysis is essentially difficult at this stage. Thus, design decisions are mostly based on DC analysis of uniform mesh structures, with current drains modeled using simple area-based calculations.<sup>1</sup> In current practice, designers

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<sup>1</sup>In later stages, with more design information available, the design may be

often try different combinations of wire pitch and width for different layers, and select the best combination based on circuit simulations [5]. Due to the time-consuming nature of circuit simulation, it is computationally infeasible to explore all possible configurations; the result is hence a suboptimal solution. In this context, it is both practically useful and theoretically interesting to seek a new approach to optimize topology for a hierarchical, uniform power distribution.

In this paper, we study the worst-case static IR-drop on hierarchical power meshes using both analytical and empirical methods. We propose a novel and efficient method for optimizing worst-case static IR-drop on hierarchical, uniform power distribution meshes. Our results can be used for planning of hierarchical power meshes in early design stages. Main contributions of our work include:

- We derive a closed-form approximation for the worst-case IR-drop on a single-level power mesh. The formula shows that given a fixed total routing area, the worst-case IR-drop increases logarithmically with the number of metal lines on the mesh.
- Based on the previous analysis and empirical studies, we propose a model for the worst-case IR-drop on a two-level power mesh, and obtain an accurate empirical expression.
- We present a new approach to optimize the topology of two-level power mesh, using the above expression. For a fixed total routing area, we can minimize the worst-case IR-drop. Alternatively, given a desired IR-drop tolerance, we can minimize the routing area used by the power mesh to achieve the IR-drop specification.
- We extend our study to three-level power meshes, and find out that more levels for the hierarchical power mesh do not significantly improve the worst-case IR-drop. The third, middle-level mesh helps to reduce IR-drop by only a relatively small extent (about 5%, according to our experiments).

**Related previous works.** Most previous works on power/ground network [8, 9, 10, 13, 14] focus on the *power network wire sizing problem*. Many mathematical programming methods are adopted to solve the problem, such as conjugate-gradient method [3], Sequential LP method [10], Incomplete Cholesky Decomposition Conjugate Gradient method [14], and multigrid-based method [13]. [2, 3, 4] discussed how to optimize the topology of the power distribution network. In particular, [2] and [3] utilize the wire sizing technique on a complete graph topology, and thus are not refined based on more accurate dynamic analysis.

capable of handling a large-scale optimization problem. [4] is based on current pattern analysis, but can only be applied to tree-like structures. However, meshes are much more robust than tree structure in terms of IR-drop variation [2]; virtually every modern design adopts a hierarchy of meshes as the basic topology for the power distribution network.

The rest of the paper is organized as follows. Section II formulates the optimization problem. Section III derives a closed-form formula for the worst-case IR-drop on a single-level power mesh. The worst-case IR-drop on two-level power meshes is discussed in Section IV. Section V discusses the optimization of two-level uniform power mesh topology. Section VI considers power meshes with more levels. Finally, Section VII gives conclusions and possible future research directions.

## II. PROBLEM FORMULATION

We focus on the following optimization problem for a hierarchical power mesh: Given fixed wire pitch and width for the bottom-level mesh, find the optimal wire pitch and width for each mesh except the bottom-level mesh, such that for a given total routing area the power mesh achieves the minimum worst-case IR-drop, or for a given worst-case IR-drop requirement, the power mesh meets the requirement with minimum total routing area.

The work is based on the following model of the power distribution network in early design stages.

- The power distribution network is constructed by a hierarchy of mesh structures connected by vias at crossing points of wires. Each mesh has uniform wire spacing and width. Ignoring the resistance of vias<sup>2</sup>, we assume perfect contact at each crossing point.
- On top of metal layers, there are arrays of C4 power pads evenly distributed on the surface of the power mesh.
- Under the bottom-level mesh, there are devices connected to the wires of the bottom-level mesh. In the early stage of design, the devices are modeled as uniform current sinks and placed at crossing points of the bottom-level mesh, since before the accurate floorplan the exact current drain at different locations is unknown.

Analytical and empirical methods are employed to solve the problem. In state-of-art designs, there are a fairly large number (> 100) of power pads evenly distributed on the surface of the top-level power mesh [15]. It is reasonable to assume that the whole power mesh is an infinite resistive grid constructed by replicating the *representative area* surrounded by four power pads. Figure 1 (a) and (b) illustrate a two-level power mesh and the *representative area*. Our analysis and circuit simulations consider only the worst-case IR-drop on the representative area. This method is also used in [5].

## III. WORST-CASE IR-DROP ON SINGLE-LEVEL POWER MESH

Static IR-drop on a hierarchical power mesh largely depends on the top-level mesh since usually the top-level mesh is wider

<sup>2</sup>Via resistance is ignored, since it is much smaller than that of mesh segments.

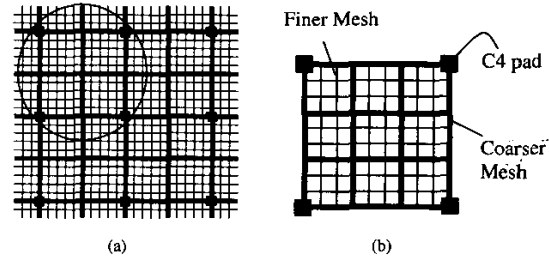


Fig. 1. A two-level power mesh and the representative area.

and coarser, and most current flows along the top-level mesh. In this section, we analyze the worst-case static IR-drop on a single-level power mesh with a fixed total routing area. The top-level power mesh is abstracted as a uniform infinite resistive lattice with edge resistance  $R$ .<sup>3</sup> C4 power pads are evenly distributed on the mesh, and we examine IR-drop in an  $N \times N$  representative area surrounded by four power pads<sup>4</sup>; the worst-case IR-drop appears at the center of this representative area. Each power pad supplies a current  $I = N^2 i$  to the power mesh, where  $i$  is the current drain at each intersection on the mesh.

Assume there is a coordinate system with the origin at the center of the power mesh. We analyze the voltage drop between the node  $(0, 0)$  and the power pad at  $(\frac{N}{2}, \frac{N}{2})$  by considering incoming currents from power pads and evenly distributed current sinks separately.

### A. IR-Drop Relative to Power Pad Locations

Suppose only a current  $I$  enters the resistive lattice at the node  $(m_s, n_s)$ . We analyze the IR-drop by first adopting an approximation for the infinite lattice, due to [11]. According to [11], the voltage drop between two nodes  $(m_s, n_s)$  and  $(m, n)$ , denoted as  $V_{(m_s, n_s)}(m, n)$ , is given by the integral

$$(IR/2\pi) \int_0^\pi (1 - e^{-|m-m_s|\alpha} \cos(n-n_s)\beta) / \sinh \alpha d\beta, \quad (1)$$

where  $\cosh \alpha + \cos \beta = 2$ . When  $|m - m_s|$  or  $|n - n_s|$  is large, the voltage drop  $V_{(m_s, n_s)}(m, n)$  can be approximated as

$$V_{(m_s, n_s)}(m, n) \approx \frac{IR}{4\pi} [\ln((m - m_s)^2 + (n - n_s)^2) + 2\pi c_1], \quad (2)$$

where  $c_1 = 0.51469$  is a constant.

Let  $V_{(m_s, n_s)}$  denote the voltage drop between  $(0, 0)$  and the power pad at  $(\frac{N}{2}, \frac{N}{2})$  caused by the incoming current from the power pad at  $(m_s, n_s)$ . According to Formula (2), we have

- when  $(m_s, n_s) = (\frac{N}{2}, \frac{N}{2})$ ,  $V_{(m_s, n_s)} \approx (IR/4\pi)(2 \ln N - \ln 2 + 2\pi c_1)$ ;
- when  $(m_s, n_s) \neq (\frac{N}{2}, \frac{N}{2})$ ,  $V_{(m_s, n_s)} = V_{(m_s, n_s)}(\frac{N}{2}, \frac{N}{2}) - V_{(m_s, n_s)}(0, 0) \approx (IR/2\pi) \ln \frac{D_s}{D_0}$ ,

<sup>3</sup>Note that for a uniform mesh, the edge resistance is determined only by the total routing area and is independent of the number of metal lines. With a fixed total routing area, when the number of metal lines on the mesh increases, wire pitch and wire width decrease with the same ratio, and the edge resistance remains the same.

<sup>4</sup>E.g., for the coarser mesh shown in Figure 1 (b),  $N = 3$ .

where  $D_s$  is the Euclidean distance between  $(m_s, n_s)$  and  $(\frac{N}{2}, \frac{N}{2})$ , and  $D_0$  is the Euclidean distance between  $(m_s, n_s)$  and  $(0, 0)$ . The constant  $c_2 = \sum_{(m_s, n_s) \neq (\frac{N}{2}, \frac{N}{2})} \ln \frac{D_s}{D_0}$  can be computed by a simple algorithm, which calculates the summation for currents from all power pads within a circle around the origin. As the radius of the circle increases, the summation converges to a constant  $c_2 = -1.40278$ .

Therefore, if only incoming currents are considered, the voltage drop between  $(0, 0)$  and the power pad at  $(\frac{N}{2}, \frac{N}{2})$  is

$$V_{source} = \sum_{(m_s, n_s)} V_{(m_s, n_s)} \approx \frac{IR}{2\pi} (\ln N + c), \quad (3)$$

where the constant  $c = \pi c_1 - \ln 2/2 + c_2 = -0.1324$ .

### B. IR-Drop Due to Evenly Distributed Current Sinks

Next, we consider the voltage drop caused by current sinks at the intersections of the power mesh. If the voltage drop between  $(0, 0)$  and  $(m, n)$  is denoted by  $V_{sink}(m, n)$ , by a combination of Ohm's and Kirchhoff's Laws we have

$$\begin{aligned} V_{sink}(m-1, n) + V_{sink}(m+1, n) + V_{sink}(m, n-1) \\ + V_{sink}(m, n+1) - 4V_{sink}(m, n) = iR, \end{aligned} \quad (4)$$

which is a discrete Poisson equation. If the resistive lattice is regarded as a discrete approximation to a continuous resistive medium, we will obtain a potential function proportional to  $D^2$ , where  $D$  is the Euclidean distance from the origin. Therefore, we assume the following representation for the voltage between  $(0, 0)$  and  $(m, n)$ :

$$V_{sink}(m, n) = k(m^2 + n^2), \quad (5)$$

where  $k$  is a constant. Equation (4) then yields

$$V_{sink}(m, n) = (iR/4)(m^2 + n^2). \quad (6)$$

When only current sinks are considered, the voltage drop between  $(0, 0)$  and the power pad at  $(\frac{N}{2}, \frac{N}{2})$  is

$$V_{sink} = V_{sink}(N/2, N/2) = IR/8. \quad (7)$$

### C. Verification of Worst-Case IR-Drop

From the above analysis, we obtain the voltage drop at the center:

$$\mathbf{V} = \mathbf{V}_{source} + \mathbf{V}_{sink} \approx \frac{IR}{8} + \frac{IR}{2\pi} (\ln N + c), \quad (8)$$

where  $c = -0.1324$ .

To verify the above formula for worst-case IR-drop on the single-level power mesh, we use HSpice to simulate various power meshes with fixed total routing area and different values of  $N$ 's. Since it is a linear problem, in our experiments the resistance of each wire segment  $R$  is simply set to be  $1K\Omega$ , and the total current drain in the area  $I$  is set to be  $1mA$ . We list simulation results for  $N$  from 4 to 12 in Table I, and compare them with the estimated values from the formula. The results show that when  $N$  is larger than 4, the formula is accurate, with error less than 1%.

TABLE I  
SIMULATION RESULTS FOR WORST-CASE IR-DROP ON THE SINGLE-LEVEL POWER MESH, COMPARED TO ESTIMATED VALUES (MV).

N	IR-Drop	Estimated IR-Drop	Error
4	333.33	324.56	8.77
6	392.86	389.09	3.76
8	436.97	434.88	2.09
10	471.73	470.39	1.33
12	500.34	499.41	0.92
100	836.87	836.86	0.01

## IV. WORST-CASE IR-DROP ON TWO-LEVEL POWER MESH

A two-level power mesh is abstracted as two uniform infinite resistive lattices, with the bottom-level grid usually being much finer than the top-level one. In this section, we analyze worst-case IR-drop on two-level power mesh by considering IR-drop on the coarser and finer meshes separately.

Suppose the representative area surrounded by four power pads is comprised of an  $N_1 \times N_1$  coarser mesh and an  $N_2 \times N_2$  finer mesh.<sup>5</sup> When  $N_1$  is odd, the worst-case IR-drop on the power mesh appears at the center of this representative area. Suppose the wiring resource for the finer mesh is 1, and the edge resistance is  $R$ . Let  $r$  ( $r > 1$ ) denote the total routing area for the combined two-level power mesh. Then the routing area for the coarser mesh is  $r - 1$ , and the corresponding edge resistance of the coarser mesh is  $R/(r - 1)$ .

### A. Equivalent Single-Level Mesh

Each power pad supplies a current  $I = N_2^2 i$  to the power mesh, where  $i$  is the current drain at each intersection on the finer mesh. When the ratio of  $N_2$  to  $N_1$  is large, the coarser mesh has much less resistance than the finer mesh, and thus most of the current flows along the coarser mesh. Assume that the currents flow along an equivalent single-level coarse mesh with edge resistance  $R_e$ . According to the analysis in Section III, the worst-case IR-drop on the equivalent single-level coarse mesh can be expressed as

$$V_1 \approx \frac{IR_e}{2\pi} (\ln N_1 + c), \quad (9)$$

where  $c$  is a constant.

We simulate two-level power meshes using HSpice, setting the routing area of the two meshes to be the same (i.e.,  $r = 2$ ), and setting the finer mesh to be 10 times finer than the coarser one (i.e.,  $N_2 = 10N_1$ ).<sup>6</sup> The resistance  $R$  is set to be  $1K\Omega$  and the total current drain in the area  $I$  is set to be  $1mA$ . We list the worst-case IR-drop  $V$  for  $N_1$  from 2 to 11 in Table II. We also plot the  $V \sim \ln N_1$  relation in Figure 2, and the worst-case IR-drop grows roughly linearly with  $\ln N$ .

<sup>5</sup>E.g., for the two-level power mesh shown in Figure 1 (b),  $N_1 = 3$  and  $N_2 = 9$ .

<sup>6</sup>As noted earlier, static IR-drop on a hierarchical power mesh largely depends on the top-level mesh. On two-level power meshes, when  $N_2 > 10N_1$  the difference of the worst-case IR-drops is relatively small and can be ignored. Therefore, in our simulations of two-level power meshes,  $N_2$  is usually set to be  $10N_1$ .

TABLE II  
SIMULATION RESULTS FOR WORST-CASE IR-DROP ON THE TWO-LEVEL  
POWER MESH WITH SAME ROUTING AREA BETWEEN THE TOP-LEVEL  
AND THE BOTTOM-LEVEL MESHES.

$N_1$	3	4	5	6
IR-Drop (mV)	170.15	188.62	206.75	219.79
Slope	0.258	0.439	0.450	0.483
$N_1$	7	8	9	10
IR-Drop (mV)	232.42	242.32	251.96	259.91
Slope	0.479	0.492	0.488	0.495

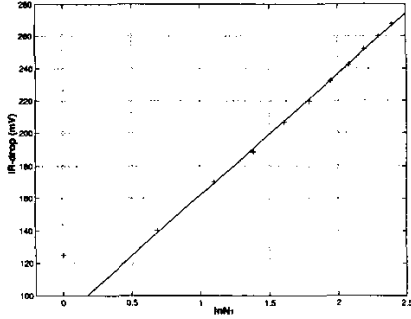


Fig. 2. Worst-case IR-drop vs.  $\ln N_1$  relation for two-level power meshes with the same wiring resource allocation between the two meshes.

The equivalent resistance  $R_e$  can be computed by calculating the slope of the line in Figure 2. The values are shown in the third column of Table II. When  $N_1 > 10$ , the  $V \sim \ln N_1$  relation is close to linear, and  $R_e$  approaches  $R/2 = R/r$ . Additional simulations confirm this relationship for different total amounts of routing area  $r$ . Table III lists the ratio of  $R$  to  $R_e$  for different  $r$ 's when  $N_1 = 19$ . According to the simulation results, the ratio of  $R$  to  $R_e$  is close to  $r$ , and  $R_e$  can be approximated by  $R/r$ . Note that when  $r$  increases, the error grows larger. The data show that although the voltage drop is largely proportional to  $\ln N_1$  when  $N_1$  is large, there are still other terms which depend on  $N_1$  and grow with  $r$ . This will be explained in the discussion below.

TABLE III  
RATIO OF  $R$  TO  $R_e$  FOR TWO-LEVEL POWER MESHES WITH DIFFERENT  
TOTAL ROUTING AREA  $r$  WHEN  $N_1 = 19$ .

$r$	1.667	2	4	6	8
$R/R_e$	1.661	1.991	3.953	5.888	7.806

### B. IR-Drop on the Finer Mesh

Although most of the current flows along the coarser mesh, IR-drop on the finer mesh cannot be neglected, especially when  $N_1$  is small. When the ratio of  $N_2$  to  $N_1$  is large, the wire segments on the coarser mesh are much wider than those on the finer mesh, and thus the resistance of a wire segment with unit length on the coarser mesh is much smaller than that on the finer mesh. Therefore, when considering IR-drop on the finer mesh, it is reasonable to assume that the finer mesh within each

cell formed by the coarser mesh has equal voltage on the cell boundary.

TABLE IV  
SIMULATION RESULTS FOR WORST-CASE IR-DROP  $V$  ON AN  $M \times M$   
MESH WITH EQUAL VOLTAGE ON THE BOUNDARY (mV).

$M$	2	3	4	5	6	7	8
$V$	0.50	1.13	1.67	2.60	3.43	4.66	5.79
$M$	9	10	11	12	13	14	15
$V$	7.32	8.73	10.55	12.27	14.38	16.39	18.80
$M$	16	17	18	19	20	21	22
$V$	21.11	23.91	26.41	29.41	32.31	35.39	37.58

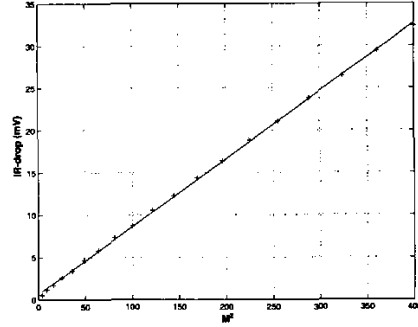


Fig. 3. Worst-case IR-drop vs.  $M^2$  relation on the fine mesh with equal voltage on the boundary (mV).

We use HSpice simulations to obtain the worst-case IR-drop  $V$  on a fine mesh with equal voltage on the boundary. In our simulations, at each node of the fine mesh, there is a current drain with  $i = 0.001mA$ , and each wire segment has resistance  $R = 1K\Omega$ . Let  $M$  denote the number of metal lines in the fine mesh. Table IV shows simulation results for different values of  $M$ . We plot the  $V \sim M^2$  relation in Figure 3. The relation displays a very clear linearity.

In the two-level power mesh, the number of metal lines of the finer mesh within each cell formed by the coarser mesh  $M = N_2/N_1$  and the total current drain within the cell  $I = N_2^2 i$ . Therefore the IR-drop in the finer mesh within a cell surrounded by the coarser mesh is roughly proportional to  $M^2 i R = IR/N_1^2$ .

### C. Formula for Worst-Case IR-Drop on Two-Level Mesh

Based on the above discussions, we seek an expression for worst-case IR-drop on the two-level power mesh with the following form:

$$V \approx \frac{IR}{2\pi r} \ln N_1 + C_1(r)IR + C_2(r) \frac{IR}{N_1^2}, \quad (10)$$

where  $C_1(r)$  and  $C_2(r)$  are functions of  $r$ .

We simulate various two-level power meshes with total routing area  $r = 16$  and different wire pitches for the coarser mesh, to compute worst-case IR-drops. In our simulations, the finer mesh is 10 times finer than the coarser one (i.e.,  $N_2 = 10N_1$ ),

the resistance  $R$  is  $1K\Omega$  and the total current drain in the area  $I$  is  $1mA$ . We list the results in Table V.

TABLE V  
SIMULATION RESULTS FOR WORST-CASE IR-DROP ON TWO-LEVEL POWER MESHES WITH TOTAL ROUTING AREA  $\tau = 16$ , COMPARED TO APPROXIMATED VALUES FROM EQUATION (10) (mV):

$\tau$	$N_1$	IR-Drop	Approx. IR-Drop	Error
16	1	77.15	82.34	5.19
16	2	29.37	32.52	3.15
16	3	26.04	26.05	0.01
16	4	<b>24.67</b>	<b>25.24</b>	<b>0.56</b>
16	5	25.76	25.75	-0.01
16	6	26.42	26.64	0.23
16	7	27.62	27.62	0.00
16	8	28.46	28.58	0.12
16	9	29.51	29.51	0.00

From the simulation results for  $N_1 = 7$  and  $N_1 = 9$  (in some sense, the choice of these two  $N_1$  values is arbitrary, although larger values are better),  $C_1(\tau)$  and  $C_2(\tau)$  can be estimated, yielding  $C_1(16) = 0.006719$  and  $C_2(16) = 0.0759$ . We can then estimate values of  $V$  for other  $N_1$ 's according to Equation (10), and compare the values with experimental results in the fifth column of Table V. The expression is accurate when  $N_1$  is odd and larger than 1, with error less than 1%. When  $N_1$  is even, worst-case IR-drop does not appear at the center of the representative area as when  $N_1$  is odd, and the errors for even  $N_1$ 's are somewhat larger.

## V. OPTIMAL PLANNING OF TWO-LEVEL POWER MESH

In this section, we consider objectives mentioned in Section II, and optimize the topology of two-level power meshes based on the above expression for worst-case IR-drop on two-level power meshes (Equation (10)).

### A. Optimizing Topology with a Given Total Routing Area

First, we optimize wire pitch and width for the coarser mesh, such that for a given total routing area  $\tau$ , the power mesh achieves the minimum worst-case IR-drop.

For a given value of  $\tau$ , to obtain the minimum value of  $V$  in Equation (10), we set the derivative of  $V$  to zero:

$$\frac{\partial V}{\partial N_1} = \frac{IR}{2\pi\tau} \frac{1}{N_1} - C_2(\tau) \frac{2IR}{N_1^3} = 0$$

Then, the optimal wire pitch for the coarser mesh, i.e. the optimal  $N_1$ , is given by

$$N_1 = \sqrt{4\pi\tau C_2(\tau)} \quad (11)$$

In practice, we can estimate the value of  $C_2(\tau)$  in the expression by simulating two-level power meshes with total routing area  $\tau$  for several values of  $N_1$ . (In the example above (Section C), for a given total routing area  $\tau = 16$ ,  $C_2(\tau)$  was estimated from simulations of power meshes for  $N_1 = 7$  and  $N_1 = 9$ .) We may then compute the optimal  $N_1$  according to Equation (11); e.g., we obtain  $N_{opt}(16) = 3.9$ , which matches simulation results in Table V.

TABLE VI  
MINIMAL WORST-CASE IR-DROPS (mV) FOR DIFFERENT VALUES OF  $\tau$ .

$\tau$	$C_1(\tau)$	$C_2(\tau)$	$N_{opt}(\tau)$	$V_{opt}(\tau)$
10	0.0768	0.010986	3.1	37.0
11	0.0766	0.009934	3.3	34.2
12	<b>0.0765</b>	<b>0.009066</b>	<b>3.4</b>	<b>31.9</b>
13	<b>0.0763</b>	<b>0.008338</b>	<b>3.5</b>	<b>29.9</b>
14	0.0762	0.007718	3.7	28.2
15	0.0761	0.007184	3.8	26.6
16	0.0759	0.006719	3.9	25.2

### B. Optimizing Topology with a Given Worst-Case IR-Drop Requirement

A second problem is to find the optimal wire pitch and width for the coarser mesh, such that for a given worst-case IR-drop requirement, the power mesh meets the requirement with minimum total routing area  $\tau$ . According to Equation (11), for each value of  $\tau$ , the minimal worst-case IR-drop occurs when  $N_1 = \sqrt{4\pi\tau C_2(\tau)}$ . Plugging into Equation (10), we obtain the minimal worst-case IR-drop for a given total routing area  $\tau$ :

$$V_{opt}(\tau) \approx \frac{IR}{2\pi\tau} \ln(\sqrt{4\pi\tau C_2(\tau)}) + C_1(\tau)IR + \frac{IR}{4\pi\tau}. \quad (12)$$

For each value of  $\tau$ , we simulate the two-level power meshes with total routing area  $\tau$  for several values of  $N_1$ , compute the values of  $C_1(\tau)$  and  $C_2(\tau)$ , and then compute the optimal worst-case IR-drop  $V_{opt}(\tau)$ . We increase the total routing area  $\tau$  until the corresponding minimal worst-case IR-drop  $V_{opt}(\tau)$  satisfies the given requirement.

For example, given a worst-case IR-drop requirement  $V > 30mV^7$ , we simulate two-level power meshes with different  $\tau$ 's for  $N_1 = 7$  and  $N_1 = 9$ , and compute the minimal worst-case IR-drop  $V_{opt}(\tau)$ . The results are summarized in Table VI. The results show that the optimal  $\tau^*$  falls between 12 and 13, and the optimal  $N_1^*$  is 3 or 4. Further simulations may be needed to get more accurate results, e.g., simulations with  $\tau \in [12, 13]$  and  $N_1 = 3$  or 4. However, we believe that this resolution in the estimation is already valuable for planning purposes.

## VI. WORST-CASE IR-DROP ON THREE-LEVEL POWER MESH

After considering two-level power meshes, it is natural to ask how much we can gain by adding more levels to the power mesh. For a three-level power mesh with a given topology (fixed number of metal lines for three levels of meshes) and a given total routing area, using *power network wire sizing* techniques, we can find the optimal *resource distribution*. For given numbers of metal lines in the top-level and bottom-level meshes, we can explore different numbers of metal lines for the middle-level mesh, and for each topology we find the optimal resource allocation solution and the corresponding worst-case IR-drop.

<sup>7</sup>Note that the required value is related to specified values of resistance  $R$  ( $R = 1K\Omega$ ) and total current drain  $I$  ( $I = 1mA$ ) in our simulations.

TABLE VII  
WORST-CASE IR-DROP OF THREE-LEVEL POWER MESHES WITH FIXED  
BOTTOM-LEVEL MESH.

$N_1$	$N_2$	IR-Drop (mV)	$r_1$	$r_2$
3	4	35.8	6.43	3.57
3	6	35.2	6.54	3.46
3	10	34.3	6.76	3.24
3	15	34.3	6.88	3.12
3	20	34.7	6.97	3.03
3	40	35.3	7.07	2.93
3	60	36.1	8.05	2.95
4	5	36.4	5.57	4.43
4	6	35.9	6.13	3.87
4	10	35.2	6.44	3.56
4	15	35.1	6.51	3.49
4	20	35.8	6.77	3.23
4	40	36.4	6.99	3.01
4	60	37.1	7.48	2.52

A key element in our exploration is the sequential LP method proposed by Tan et al. [10]. Their method assumes that the current direction on each branch is known, and then minimizes total wiring area under maximum worst-case IR-drop and maximum branch current constraints. The optimal sizing problem here is much easier than Tan's formulation, because we only need to decide the values of three variables, namely, the wire widths of uniform meshes at three different levels. From the symmetric structure of three-level meshes, we already know which node on the power mesh has the worst-case IR-drop. These two advantages allow us to further simplify the original method to accelerate the search.

In our scheme, for a given width assignment, we can find the voltage at each node by solving a set of linear equations. We then fix the node voltages and find the optimal width assignment to maximize current drain under the node where worst-case IR-drop occurs. After obtaining a new wire width assignment from the linear programming, we repeat this process iteratively until the solution converges. The algorithm is implemented using the linear programming package from IBM Optimization Solutions Library [16]. Our implementation allows us to explore the optimal wire width assignment for a three-level mesh with more than 100,000 nodes.

In the experiments, we fix the routing resource of the bottom-level  $120 \times 120$  mesh to be 1, and set the total routing resource of the top-level and middle-level meshes to be 10. We find the optimal routing resource distribution between the top-level and middle-level meshes using the algorithm. Table VII depicts the results. When  $N_1 = 3$ , with different middle-level meshes, the optimal worst-case IR-drop changes in the range from  $34.3mV$  to  $36.1mV$ . When  $N_1 = 4$ , the optimal worst-case IR-drops range from  $35.1mV$  to  $37.1mV$ . The middle-level mesh helps to reduce IR-drop only to a relatively small extent (about 5%).

## VII. CONCLUSION AND FUTURE WORK

In this paper, we have discussed static IR-drop on hierarchical uniform power meshes. Based on analytical and empirical

studies, we have obtained accurate expression for worst-case IR-drop on two-level power meshes. With the expression, we can optimize topology of two-level power mesh, so that for a given total routing area, the worst-case static IR-drop is minimal, or for a given IR-drop tolerance the power mesh achieves the IR-drop specification with minimal total routing area. Our approach can be used in early design stages to decide nominal wire width and pitch for power networks.

Further research directions may include: (1) optimization of non-uniform power meshes; and (2) interactions with layout or detailed current analysis.

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